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THE PEAK GAIN AND SYSTEM PERFORMANCE OF A  
LARGE PARABOLOIDAL ANTENNA

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-64-132

NOVEMBER 1964

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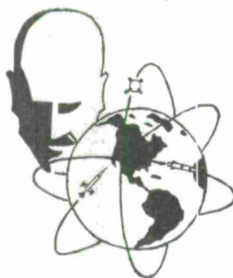
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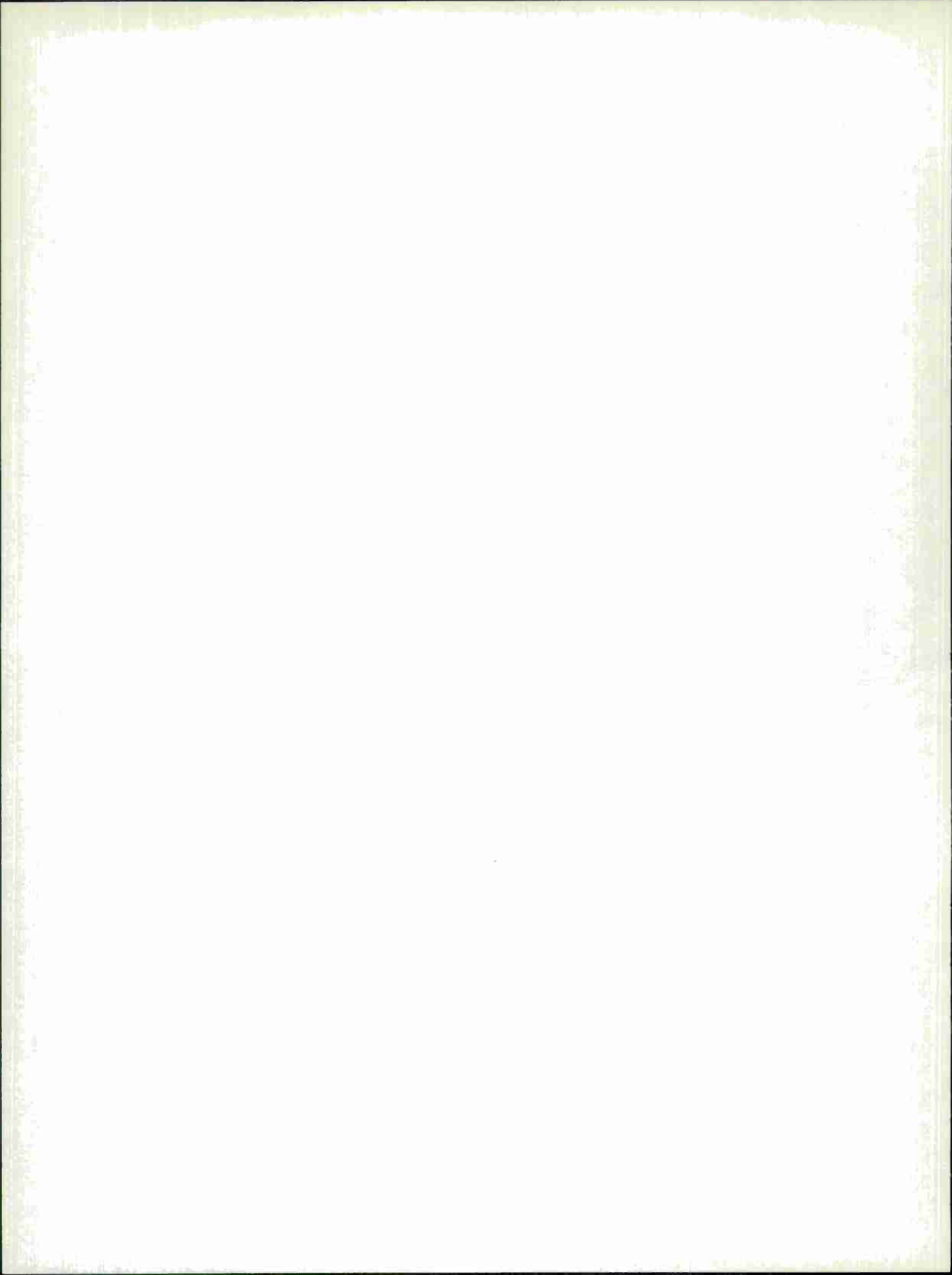
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# THE PEAK GAIN AND SYSTEM PERFORMANCE OF A LARGE PARABOLOIDAL ANTENNA

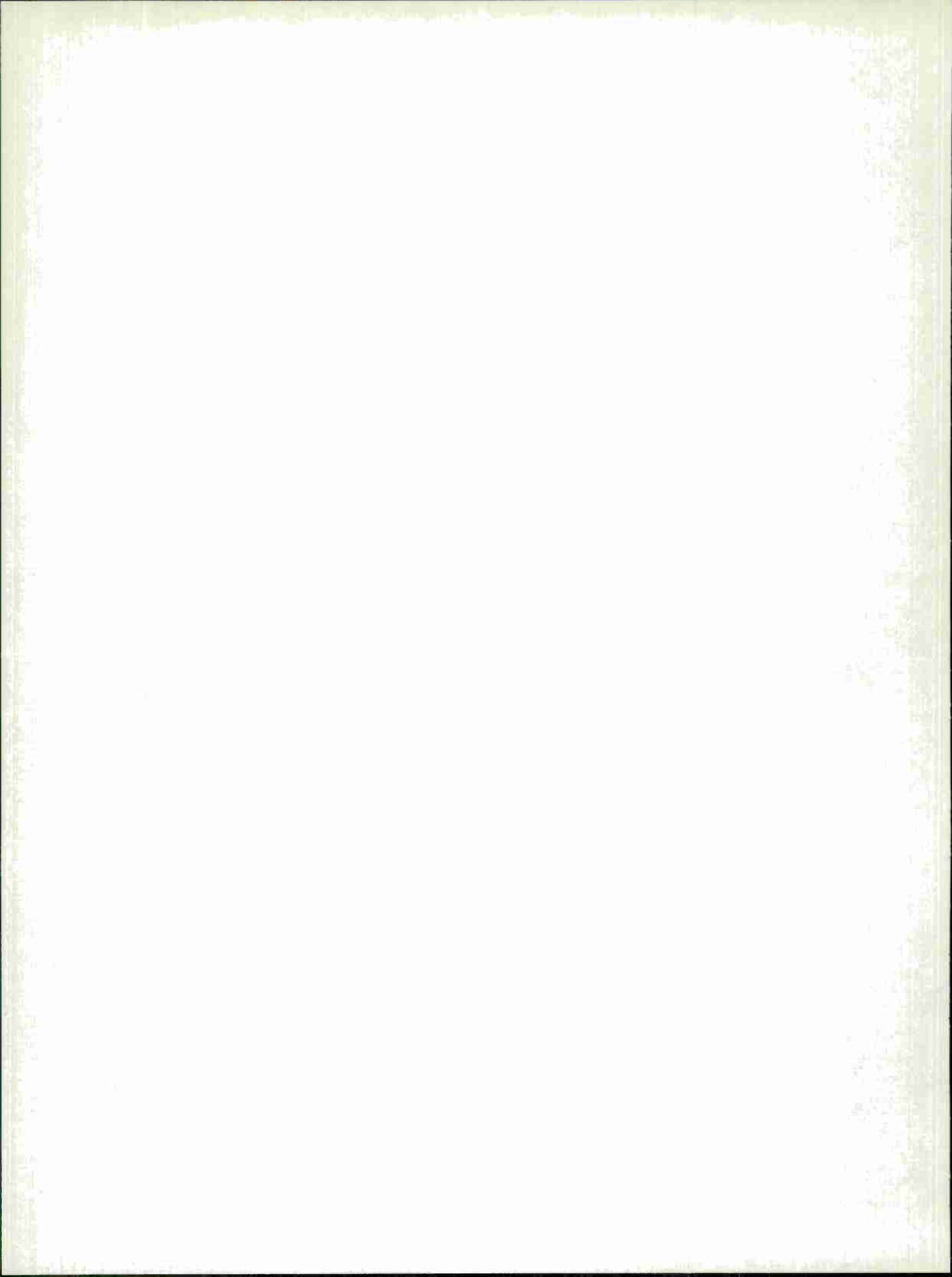
## ABSTRACT

The characteristics that affect the practical operating gain of parabolic reflector-type antennas are discussed, not for design considerations, but to analyze for the systems engineer some of the factors involved in estimating practical performance under arbitrarily set conditions. The effects of structural design, mechanical tolerances and deformations, illumination, and other considerations on the gain of large antennas operating at relatively high frequencies are examined, using theory, graphic data, and an analysis of a typical case using a conventional 60-foot telemetry antenna as an example. The possible system performance degradation resulting from the use of existing large antennas in the new 2200 to 2300-mc telemetry band is discussed.

## REVIEW AND APPROVAL

This technical documentary report has been reviewed and is approved.

*for*   
ROY D. RAGSDALE  
Colonel, USAF  
Director, Aerospace Instrumentation



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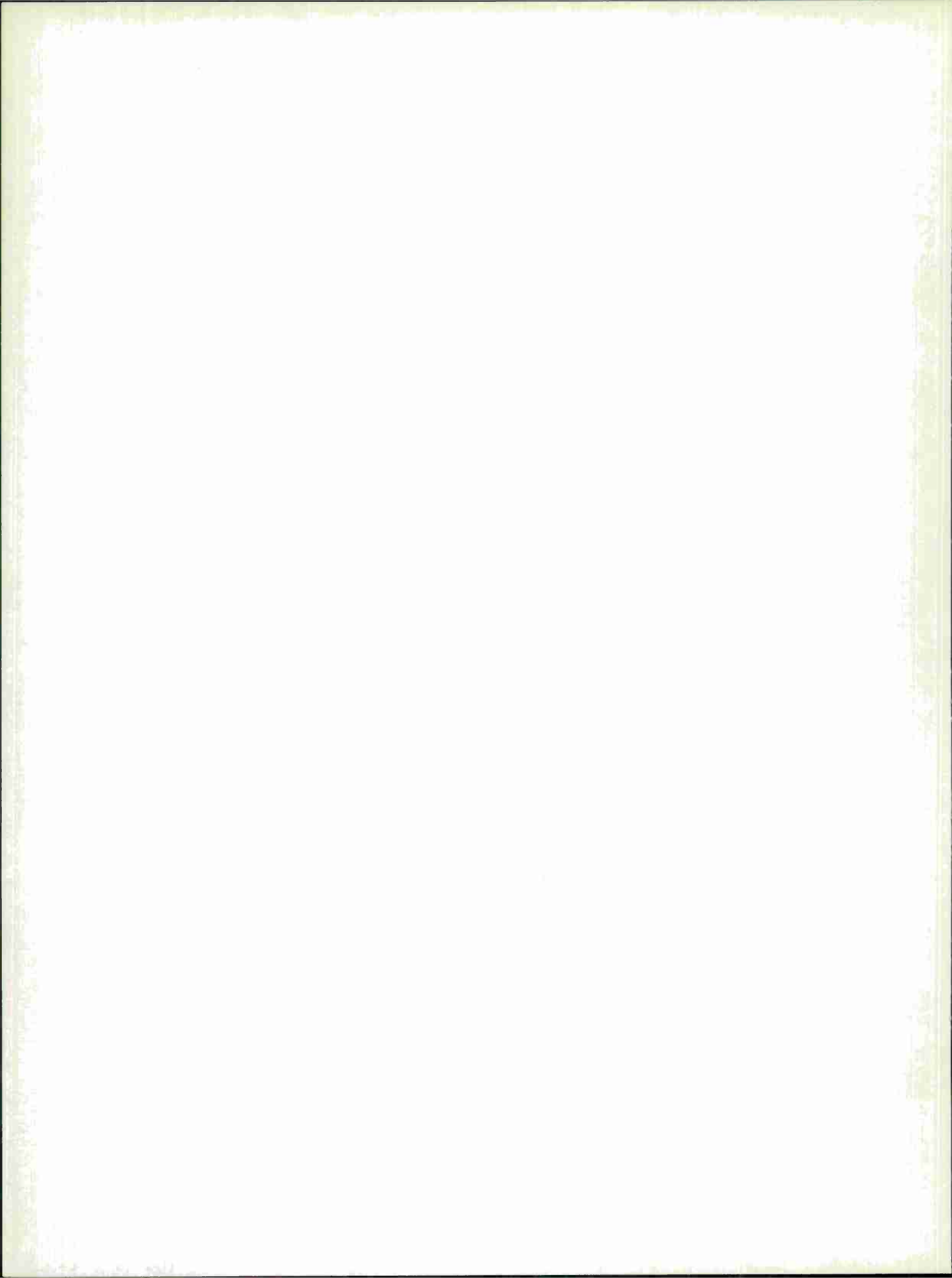
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# THE PEAK GAIN AND SYSTEM PERFORMANCE OF A LARGE PARABOLOIDAL ANTENNA

## I GENERAL INTRODUCTION

This work was originated after participation at AMR<sup>\*</sup> in the ARIS 3<sup>\*\*</sup> Telemetry Antenna discussions. The basis of these discussions centered on the advantages and disadvantages of replacing a 40-foot dish with a 60-foot dish in order to provide better telemetry coverage of certain space programs. The theoretical advantage is 3.6 db, but there is so much controversy about the practical operating gain figure of any dish, that it was believed advisable to clarify this important system design factor; especially since even the theoretical increase in gain still would give a marginal system. (The ship-board fitting of even the 40-foot dish had proved to be no easy task, and an additional cost of at least \$3 million plus serious time-slippage would be involved with the 60-foot dish.)

We believe that the 3.6-db theoretical advantage would be substantially reduced, in practice, because of certain corrections to the nominal gain figure that apply to a practical design, in addition to the more obvious operating losses with small elevation angles and the normal deterioration in performance with time and environmental stresses. These corrections may be termed the "frequency-dependent corrections," caused by mechanical tolerances and deformations. They are particularly important for large antennas, antennas used at high frequencies, and for broad band antennas. It was in the area of

---

\* Atlantic Missile Range.

\*\* Advanced Range Instrumentation Ship.

operations at higher frequencies, using standard, large, telemetry antennas, that we believed the theoretical 3.6-db margin of the ARIS case might be lost (see page 35).

It is intended that this report be part theoretical and, by including data from operational antenna tests, part practical. However, no data on operational tests was available from the Divisions and Centers, and requests for Technical Specification and Acceptance Test procedures also did not produce a sufficient amount of data for inclusion. It was ascertained that USAF operates 80 percent of all the large parabolic telemetry antennas in use in the free world, and that little documentation is available to certify the overall performance of these antennas. Since acceptance tests and operational data are unobtainable, further delay in publishing this report was deemed inadvisable.

We believe that some discussion of the basic factors governing the performance of large parabolic antennas would be helpful before discussing the detailed factors that can contribute to a substantial degradation of performance. For this reason, this document is divided into two sections, one dealing with a general discussion of antenna performance, and the other containing, in more detail, the theory and a graphical approach for determining antenna gain.

The General Discussion, Section II, is in simple terms to enable the nonspecialist to understand better the specialized terminology with which this technical field is burdened. Section III, on the determination of performance, contains a series of graphs and a procedure, both derived from the mathematical appendices.

The particular parameters of interest are the antenna gain, radiation pattern structure, and signal-to-noise (S/N) ratio that may be expected from a

practical design, together with their effects on the system costs. This report does not deal with the S/N aspect, which is highly dependent on siting and elevation angle.

For the purpose of this report, a large parabolic antenna is one with a diameter of more than 30 feet, since the practical issue is whether large antennas are worth their cost. From an engineering viewpoint, this report shows there are substantial difficulties in maintaining the theoretical gain advantage of large antennas, especially at high frequencies, and that, from a cost point of view, there must be an upper limit.\*

The discussion in this report is limited to illumination of the reflector by a single source situated on, and having a radiation pattern substantially symmetrical about, the reflector axis. In addition, design and use details, such as cross polarization, multiple reflections between the parabola and feed, and feed structure imperfections, are not specifically illustrated. Similarly, dissipative losses, such as those in the feed transmission line, radome, or reflecting surfaces, are not discussed.

---

\* The dollar cost of a conventional, steerable parabolic antenna system has been quoted as  $5 (D)^{2.7}$ , so that a 60-foot dish installation costs about \$315,000 (see Reference 1).



## II GENERAL DISCUSSION OF FACTORS AFFECTING PEAK GAIN

The purpose of this section is to explain the performance of a parabolic antenna in simple terms, so that the specialized terminology and special details that contribute to a practical gain figure may be more easily understood. The approach used is a combination of optics, diffraction theory, and simple antenna radiation patterns, all of which are quite well known or can be readily verified.

### APERTURE, RADIATION PATTERN, AND THEORETICAL (APERTURE) GAIN

Figure 1 shows a section through a parabolic surface, PP; XX is the axis of symmetry, and FF is a line normal to the axis that passes through point,  $f$ , known as the focus, on the axis. The outstanding property of the

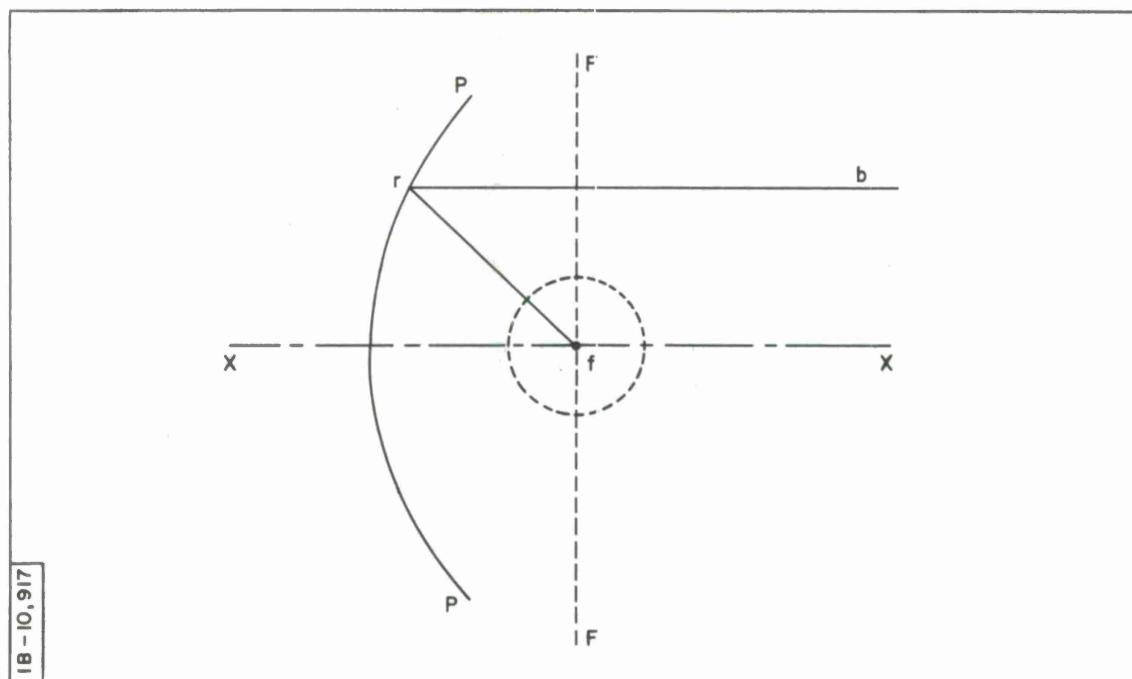


Fig. 1. Section Through a Parabolic Surface



parabolic surface is that any ray path from  $f$  to  $r$  is reflected from the surface along a path,  $rb$ , that parallel to the axis, and the length of the portion from  $f$  to the focal plane (a plane through  $FF$  and normal to the paper), or to any other parallel plane, is constant, regardless of the initial angle of the portion  $fr$  to the axis.

At optical frequencies, a source of energy placed at the focus, with a circular radiation pattern,<sup>\*</sup> as indicated by the dotted circle in Fig. 1 will be reflected from the parabola as a cylindrical beam parallel to the axis  $XX$ .

At "radio" frequencies, the wavelength of the energy is sufficiently comparable to the reflector dimensions, so that the reflected beam is no longer parallel to the axis  $XX$ . This is so even if the radiation pattern of the source is circular (in the plane  $XX$ ,  $FF$ ), and even if the wavefront in the perimeter plane,  $PP$ , is of constant intensity and in phase at all points. (The perimeter plane is the aperture plane, and that portion enclosed by the perimeter is known as the aperture.)

For these conditions, and using the sufficiently accurate Kirchhoff diffraction method,<sup>[2]\*\*</sup> the ratio of the field strength at any point in the reflected beam to that in the aperture plane is given by:

$$\begin{aligned} \text{Field Strength Ratio} &= \frac{r}{2d} \frac{(1 + \cos \theta)}{\sin \theta} J_1 \left( \frac{2\pi r}{\lambda} \sin \theta \right) \\ &= \frac{r}{2d} \frac{J_1 \left( \frac{2\pi r}{\lambda} \sin \theta \right)}{\tan \theta/2} \end{aligned} \quad (1)$$

---

\* Equal intensity in all directions from  $f$ .

\*\* Numbers in brackets designate References.



where

$2\pi r$  is the perimeter of the aperture,

$d$  is the distance from the center of the aperture to the point

$\theta$  is the angle that the distance line makes with the axis  $XX$ , and

$J_1$  is the Bessel function of the first order. [3, 4, 5, 6, 7]

For any significant angle,  $\theta$ , the radiation pattern denoted by Eq. (1) depends primarily on the Bessel function variation. This function,  $J_1(m)$ , has zeros at  $m = 3.83, 7, 10.2$ , etc. (see Fig. 2), so that the field strength ratio also has zeros, or nulls, when  $2\pi r/\lambda \sin \theta$  has these values. Between these nulls, the field strength ratio peaks to maxima, that is, the radiation pattern has lobes; but, for the larger angles, these maxima (known as side lobes) are progressively reduced in magnitude. The net result is a radiation pattern as illustrated in Fig. 3, in which the first null on each side of  $XX$  is placed at  $2\pi r/\lambda \sin \theta = 3.83$  (see also specimen pattern, Fig. 4). This gives a beamwidth for the central lobe of  $70 \lambda / r$  degrees between the first nulls. (The practical useful beamwidth is less than this, of course.)

By squaring Eq. (1) and dividing the result by  $(r/2d)^2$ , we can determine the gain; the peak value along the axis,  $XX$  is given by:

$$\text{Theoretical Peak Gain}^* = \frac{4\pi A}{\lambda^2} = \left(\frac{\pi D}{\lambda}\right)^2 \quad (2)$$

where

$A$  is the aperture area, and

$D$  is the aperture diameter.

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\*This value is conventionally known as "Directivity," or "Directive Gain."

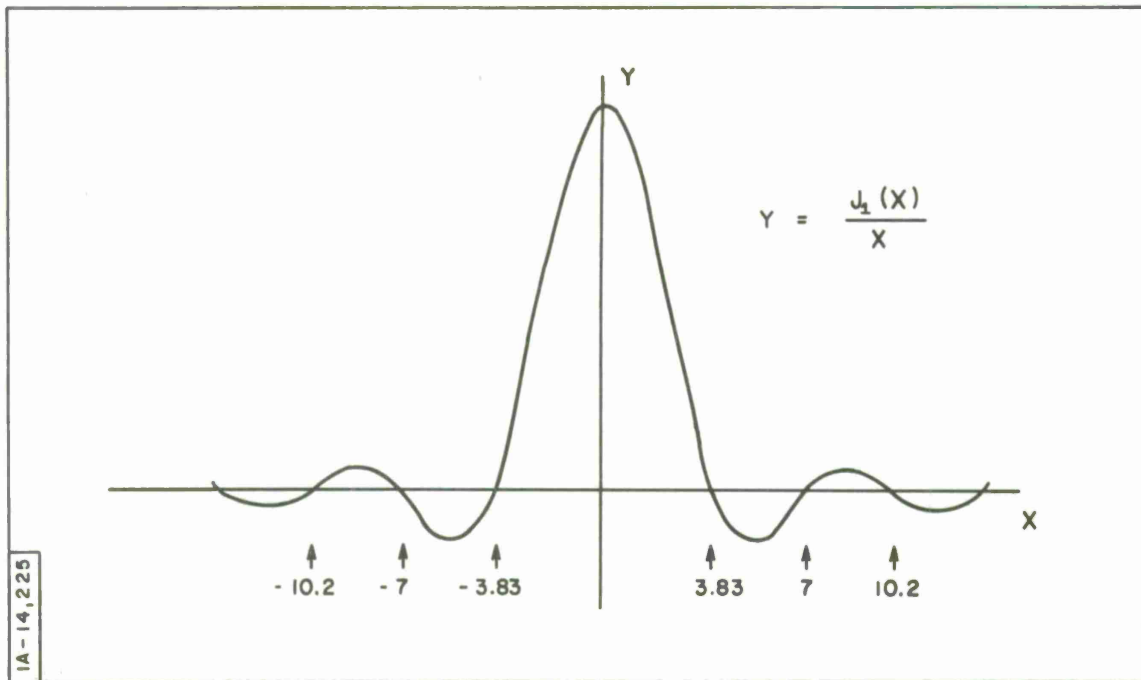


Fig. 2. Rectangular Plot of  $J_1(X)/X$

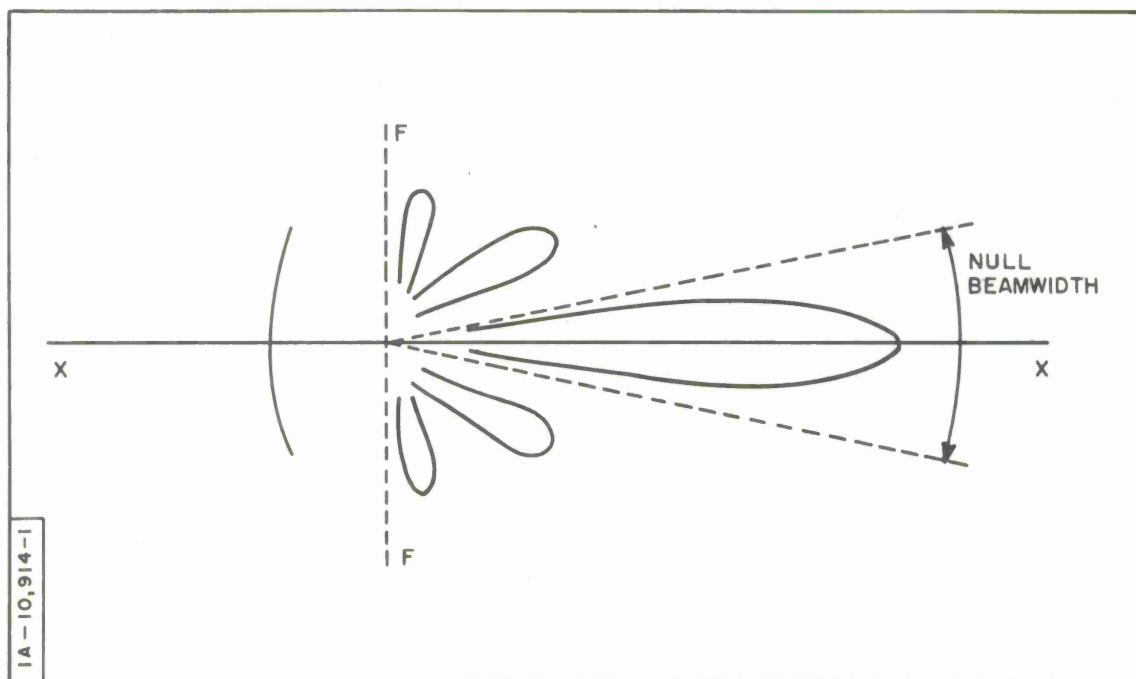


Fig. 3. Polar Plot of  $|J_1(X)/X|$

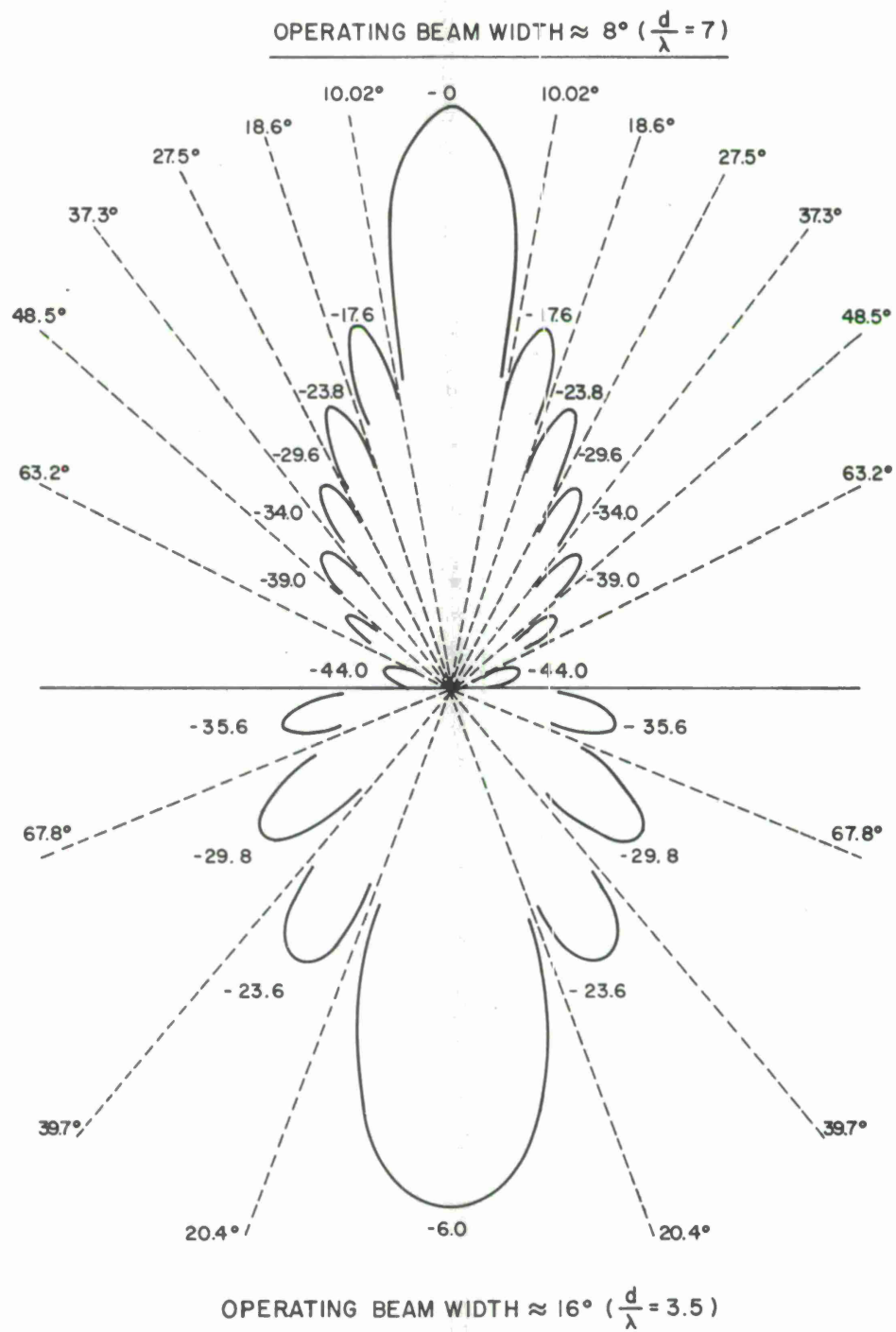


Fig. 4. Theoretical Antenna Patterns

This gain is often known as the aperture gain, since it is calculated on the assumption that the aperture wavefront is of equal intensity and phase at all points in the aperture plane. Any other condition gives less peak gain for a given power density (power per unit area) across the aperture.

#### APERTURE FIELD DISTRIBUTION, APERTURE TAPER, AND ILLUMINATION FACTOR

The above discussion assumes that

- (a) the intensity of the wavefront across the aperture, i.e., within the perimeter plane, is constant at all points, and
- (b) the phase of the aperture wavefront is the same at all points.

Because of the geometrical properties of the parabola explained in Fig. 1, the fact that electromagnetic energy travels in straight lines, and if the source at the focus is not too large, makes it possible to maintain the same phase at all points in the aperture wavefront (subject only to manufacturing precision).

At "radio" frequencies, however, there is no source whose radiation pattern is spherical, although the concept is often used as a reference and is called an isotropic source. As a simple example, the case of the half-wave dipole will be considered. Its radiation pattern resembles the surface of a doughnut, so that in a plane normal to the dipole it is circular, (Fig. 5), but in the plane through the dipole it is as shown in Fig. 6. In this latter plane, then, a dipole used as the source at the focus of a parabola will cause the field intensity across the aperture to reduce towards the edges of the aperture. The aperture field distribution will therefore vary as shown in Fig. 7, and the ratio of the axial to edge power intensities, expressed in decibels, is known as the aperture taper.

While the dipole is often used as a source, that fraction of its radiation that does not illuminate the aperture is wasted, and also contributes to

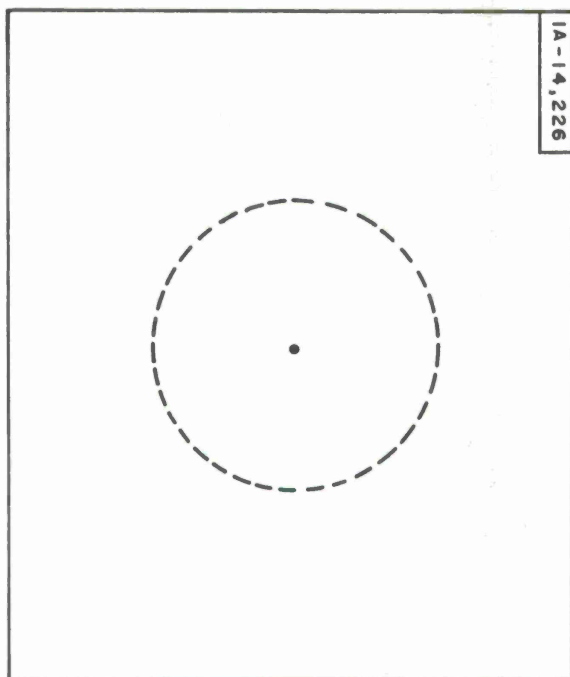


Fig. 5. Dipole Radiation Pattern in a Plane Normal to the Dipole

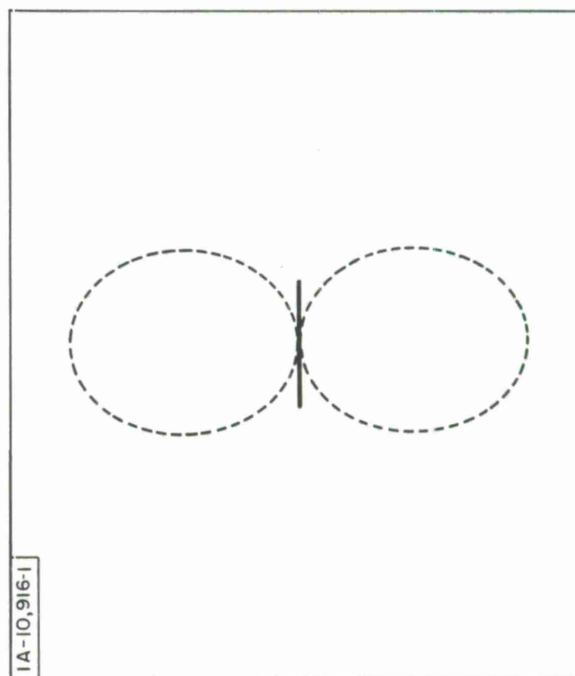


Fig. 6. Dipole Radiation Pattern in a Plane Containing the Dipole

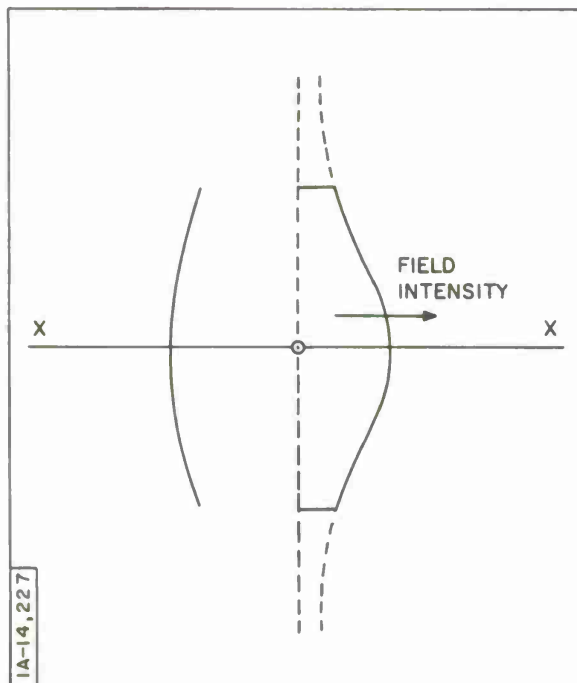


Fig. 7. Aperture Field Distribution of a Parabola Illuminated by a Dipole

interference with the formation of the desired reflected beam. To avoid this loss, a reflector (e. g. , another half-wave rod, or a hemispherical reflector of half-wave radius) can be fitted behind the dipole source; or, more commonly at higher frequencies, a horn feed can be used. All of these devices, while they direct more power into the parabola, have their own deficiencies. No directionality can ever be imparted to an antenna that has dimensions comparable to the wavelength without producing a multilobed radiation pattern, as was seen in the case of a parabola illuminated by the "ideal" source having a spherical radiation pattern. Hence, the practical design of feeds for a parabolic antenna is a matter of compromise. The illumination factor is defined as the fraction of the source power intercepted by the aperture.

Because the source to reflector path length varies from the center to the edge, there is also "space attenuation" that provides additional taper effect (see Amplitude Distribution, page 16).



## MECHANICAL PROBLEMS OF PARABOLIC ANTENNAS

### Aperture Block

In designing the source, or feed, for a parabolic antenna so as to optimize the aperture field distribution and maximize the aperture illumination, there is a tendency for the source to become large enough so that an appreciable fraction of the center of the reflected beam is blocked. This is known as aperture block, and can be quite significant for parabolic antennas having multiple feeds for operation over a wide frequency band. (The supporting structure for the feed also blocks a significant fraction of the reflected beam.)

The aperture block effect can be more serious for a tapered illumination than may appear at first sight because a centrally located feed structure blocks that portion of the aperture having the greatest power density. In all cases there is a reduction in gain when the aperture is blocked. However, the gain reduction, in decibels, for a centrally located aperture block with a parabolic aperture illumination and zero edge taper is twice the reduction, in decibels, for a uniform aperture illumination.

A formula for the reduction in gain due to aperture blocking is derived in Appendix I.

### Mechanical Tolerances/Deformations and Phase Errors

This subject is dealt with in greater detail under Tolerance Considerations, pg. 18. It becomes a matter of some importance in the case of narrow beamwidth antennas, where the peak gain is high, because such antennas are usually large physically. Mechanically speaking, large structures usually have correspondingly large tolerances and large deformations under environmental stresses (such as temperature, wind, ice, snow, etc.). Additionally, large structures may be designed closer to the minimum acceptable safety margins

in order to reduce weight, cost, and the power necessary to turn them in azimuth or elevation.

Hence, the larger the antenna, the greater the probability of relatively large changes in the position of the reflecting surface with respect to the radiation source. But a phase error in the energy reflected into the main beam is dependent only on the change in the source-to-reflector path length with respect to the operating wavelength. Thus, large antennas are more susceptible to loss of peak gain by mechanical tolerances or deformations.

In general, this subject is usually analyzed on the basis of hypotheses such as

- (a) the assumption that these phase errors are of a random nature across the aperture;
- (b) the assumption that these phase errors are symmetrically circular about the antenna axis and follow some simple law with radius; and
- (c) the assumption as in (b), but with an elliptical symmetry about the antenna axis.

These hypotheses are sufficiently representative of the most probable forms of mechanical tolerance or deformation to provide good minimum estimates of the loss in peak gain that must be allowed for in a practical systems application of large antennas.



### III DETERMINATION OF THE PEAK GAIN OF LARGE APERTURE, PARABOLIC REFLECTING ANTENNAS

#### INTRODUCTION

The primary consideration in evaluating large aperture antennas is the peak gain. Even such an important consideration as side-lobe level is secondary to the achievement of the highest possible antenna gain, and is a problem for the design engineer rather than the systems engineer.

The approach to be taken is not a systematic means of designing a high gain pencil beam antenna. Rather, it is a simple means for a systems engineer concerned with realistically predicting the expected performance capabilities of large aperture parabolic reflecting antennas.

The material to be presented is a self-contained, easy-to-use, set of graphs. To this extent, it is a new approach to an old problem. The theoretical basis for the graphs is available in most standard texts, and graphical presentation of some of the material has appeared in bits and pieces, from time to time, in various technical publications and handbooks.

#### DESCRIPTION OF ANTENNA GAIN REDUCTION FACTORS

The four principal and unavoidable modifiers of the simple peak gain formula, Eq. (2), are:

- (a) non-uniform amplitude distribution,
- (b) illumination efficiency,
- (c) aperture block, and
- (d) mechanical tolerances and deformations.

In this report, we are not concerned with dissipative losses, such as eddy currents in the reflector surface, or transmission, transfer or coupling losses from the RF generator to the radiating aperture of the source. However, the systems engineer should confirm that such losses have been accounted for in his system.

#### Amplitude Distribution (Taper Factor), $\eta$

The amplitude distribution across the aperture is a tapered (non-uniform) distribution both by chance and by choice. The primary source, located at the focus of a paraboloidal reflector, is itself an antenna with its own radiation characteristics and gain pattern, and, in order to be effective, it must radiate most of its power in the direction of the reflector. Provision of radiation characteristics that will not generate a uniform amplitude distribution, and maintenance of great amounts of power outside the reflector impose impossible requirements on the taper of the primary source.

Another cause of a tapered aperture distribution is the greater spread of energy radiated toward the edges of the reflector than that radiated into the center of the reflector because of the longer path to the reflector. This effect is called "space attenuation," and, in practice, there is no way to compensate for this.

Finally, a uniform illumination is not desirable in most cases, even if it could be achieved, because this results in the highest side-lobe level relative to the peak gain. A tapered amplitude distribution is, therefore, always present, and the peak gain is reduced from that given by a uniform distribution. The fractional reduction in gain due to a non-uniform aperture distribution is termed the taper factor,  $\eta$ , and is the first correction factor modifying the peak gain of an antenna. An analytical evaluation of  $\eta$  is given in the Appendix I.

### Illumination Factor, $\alpha$

No matter how a reflecting antenna is designed, there is always some power radiated by the primary source that is not intercepted by the reflector. This is sometimes called the spillover power. Only the fraction of the power from the source that is intercepted by the reflector is effective in producing gain. This fraction is called the illumination factor,  $\alpha$ , of the antenna, and is the second factor modifying the peak gain of the antenna.

There are definite limitations on the fraction of total power that can be directed into the reflector. For example, only the power contained in the main lobe of the primary source is of any practical use. Additionally, limitations usually occur as a result of the following relation between  $\alpha$  and  $\eta$ . Because of the taper of the primary source, as more of its main lobe is intercepted by the reflector, the more the aperture distribution becomes tapered, thereby reducing  $\eta$  even though  $\alpha$  is increased. Since both  $\alpha$  and  $\eta$  modify the peak gain, the most desirable value of  $\alpha$  is the one for which the product  $\alpha$  and  $\eta$  is a maximum. Values of  $\alpha$  for paraboloids illuminated by small horns are plotted in Fig. 9 along with a plot of the  $\alpha \eta$  product. An edge taper of 10 db is shown to be close to an optimum choice.

### Aperture Block Factor, $\beta$

This is an actual physical blocking of the aperture caused by the primary source and any support structure needed to hold it rigidly in place. An aperture block factor,  $\beta$ , is assigned to the reduction of the peak gain of the antenna by this effect, and is the third factor modifying the peak gain of the antenna. A calculation of  $\beta$  is made in Appendix I, and attention is drawn to the fact that  $\beta$  is not proportional to the percentage of the physical aperture blocked.

### Tolerance Considerations

In the construction and use of an antenna, the physical dimensions of the structure cannot be maintained accurately. Even if manufacturing techniques permit a high degree of accuracy in establishing the required physical dimensions, environmental conditions to which the antenna is subjected, when operating in the field, create stresses and strains that alter the dimensions of the structure.

Small errors in the physical dimensions of the structure have a negligible effect on the amplitude distribution of the electric field across the aperture, but these dimensional errors cause phase deviations in the phase distribution across the aperture that can result in a significant reduction of peak gain. This is the fourth factor modifying the peak gain.

The significant reduction in peak gain occurs because a given change in the ray path length to or from the reflector is small compared to the path length itself, whereas it is not small compared to the operating wavelength. The amplitude change is, therefore, much smaller than the phase change. Also, since the dimensions of a reflecting antenna must always be very large compared to the operating wavelength, if the antenna is to have gain, the phase errors produced by mechanical errors are the prime cause of reduction in gain.

Three general classes of phase errors, usually considered, that have their origin in the deviation of the antenna structure from a specified set of dimensions are:

- (a) irregular phase errors, of a random nature having a root-mean square (rms) value across the plane of the aperture;

- (b) symmetrical phase errors, where the error at any point in the plane of the aperture is a function of the radial distance from the center; and
- (c) astigmatic (bi-symmetrical) phase errors, where the loci of constant phase error are ellipses in the plane of the aperture, the major and minor axes of which intersect at the center.

At relatively low frequencies, the reduction in peak gain caused by phase deviations generally constitutes a second-order correction in the calculation of peak gain when compared to the taper factor, the illumination factor, and the aperture block factor. There is no inherent characteristic in either the taper factor, illumination factor, or aperture block factor of an antenna that suggests any disadvantage to increasing the size of an antenna, but structural limitations will ultimately pose limitations on the electrical (phase) performance of the antenna, particularly at high operating frequencies. Where this limit is reached depends, to a large extent, on how much money can be spent per decibel increase in gain and, therefore, varies, depending on the particular set of circumstances involved. However, a calculation of phase errors forms a definite basis of comparison of gains between different sizes of antennas when the structural characteristics of the antenna are known and dimensional tolerances are determinable.

Curves plotting gain reduction as a function of phase error in wavelengths for the three classes of phase errors are included among the graphs. Formulas for these curves are included in Appendix II.



### First and Final Estimates of Peak Gain

Taking into account taper factor,  $\eta$ , illumination factor,  $\alpha$ , and aperture block factor,  $\beta$ , we have:

$$\text{First Estimate Peak Gain} = \eta \alpha \beta \left( \frac{\pi D}{\lambda} \right)^2. \quad (3)$$

Taking into account phase errors, we have

$$\text{Final Estimate Peak Gain} = (\gamma_1 \gamma_2 \gamma_3) \eta \alpha \beta \left( \frac{\pi D}{\lambda} \right)^2, \quad (4)$$

where

$\gamma_1, \gamma_2, \gamma_3$ , are loss factors due to various causes of phase error.

### USE OF THE ANTENNA COMPUTATION GRAPHS

It is usually convenient to perform the gain-reduction computation in two stages, making a first estimate of the nonfrequency dependent factors, and a final estimate including the frequency dependent factors.

Taper, illumination, and aperture block are nonfrequency dependent factors; random, quadratic, and bi-symmetrical phase errors are frequency dependent factors.

### A First Estimate of Peak Gain (Figures 8 through 11)

Figure 8 is a (theoretical) gain nomograph. It gives the gain, in decibels, of a circular aperture with a uniform amplitude and phase distribution as a function of frequency and aperture diameter, and uses the basic formula

$$G_0 = \left( \pi D / \lambda \right)^2 \text{ (see Appendix I).}$$

Figure 9 is a plot of taper factor  $\eta$ , and illumination factor,  $\alpha$ , and of their product (in decibels), as a function of the (given) edge taper (in decibels). The taper factor curve assumes a parabolic amplitude distribution with radius. The edge taper is the ratio of the field at the edge of the aperture to the field at the center of the aperture (in decibels). The illumination factor curve is valid specifically for paraboloids illuminated by small horns, and is sufficiently accurate for all primary sources having approximately 85 percent of their total power in the main lobe of their radiation patterns.

Figure 10 is a nomograph which gives the fraction of the aperture blocked, in decibels, as a function of aperture diameter and the area of the aperture block. Figure 11 is a plot of the aperture block factor,  $\beta$ , (in decibels) as a function of the fractional aperture block, and the ratio of blocking area to the area of the aperture, for various values of illumination taper.

A first estimate of antenna gain can be made using Figs. 8 through 11. The information needed is the aperture diameter, frequency, edge taper, and the area of the aperture that is blocked by the primary source and its supporting structure. From Fig. 8, the aperture gain  $G_0$  is found. The taper factor,  $\eta$ , and illumination factor,  $\alpha$ , are then subtracted using Fig. 9. The fractional aperture block (in decibels) is determined from Fig. 10. The aperture block factor,  $\beta$ , is then determined using Fig. 11, and is, in turn, subtracted from  $G_0$  (decibels). Thus, the first estimate of the peak gain  $G_m$  (decibels) is given by:

$$G_m = G_0 - \eta - \alpha - \beta , \quad (5)$$

where all quantities are expressed in decibels.

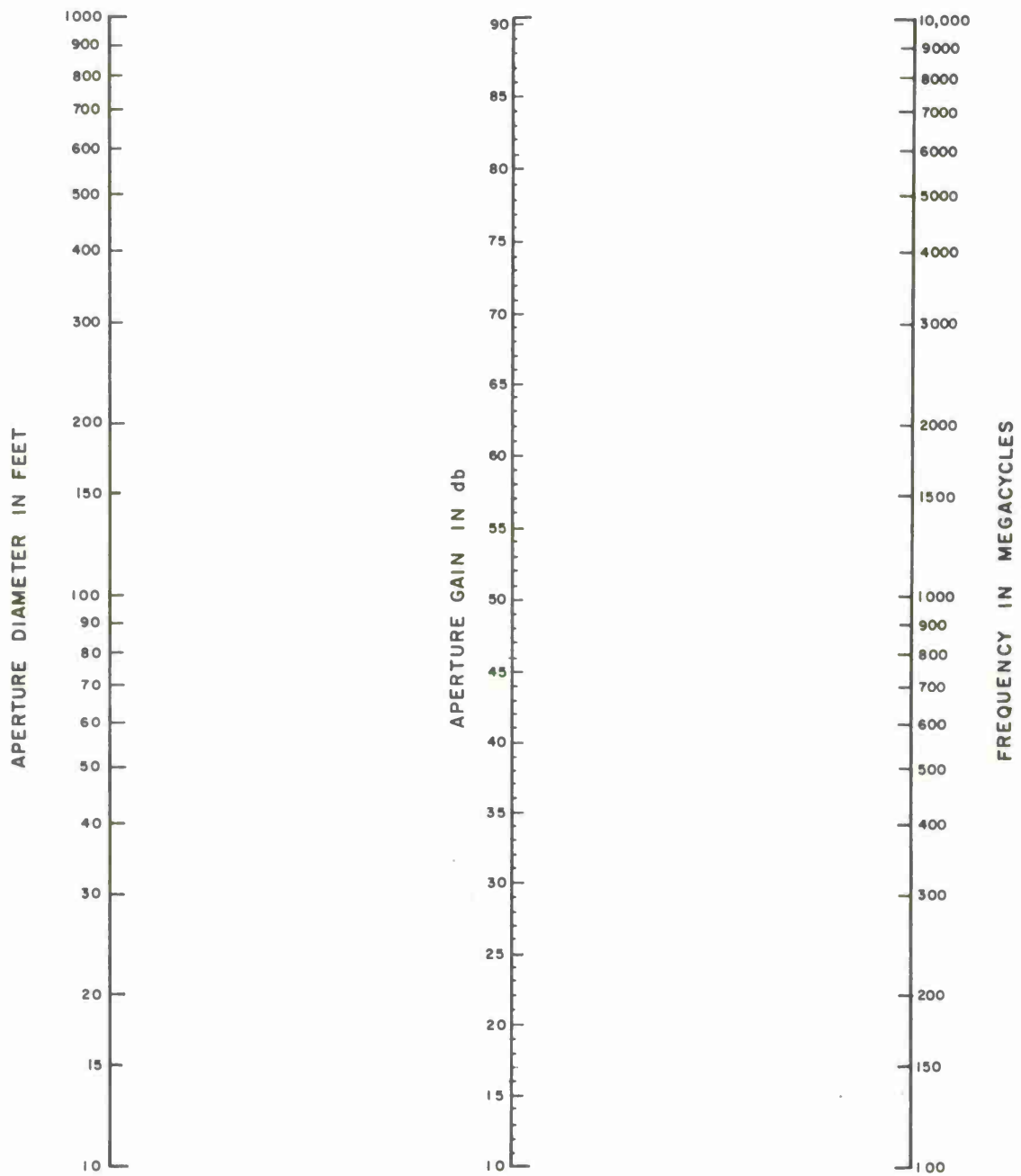


Fig. 8. Aperture Gain Nomograph



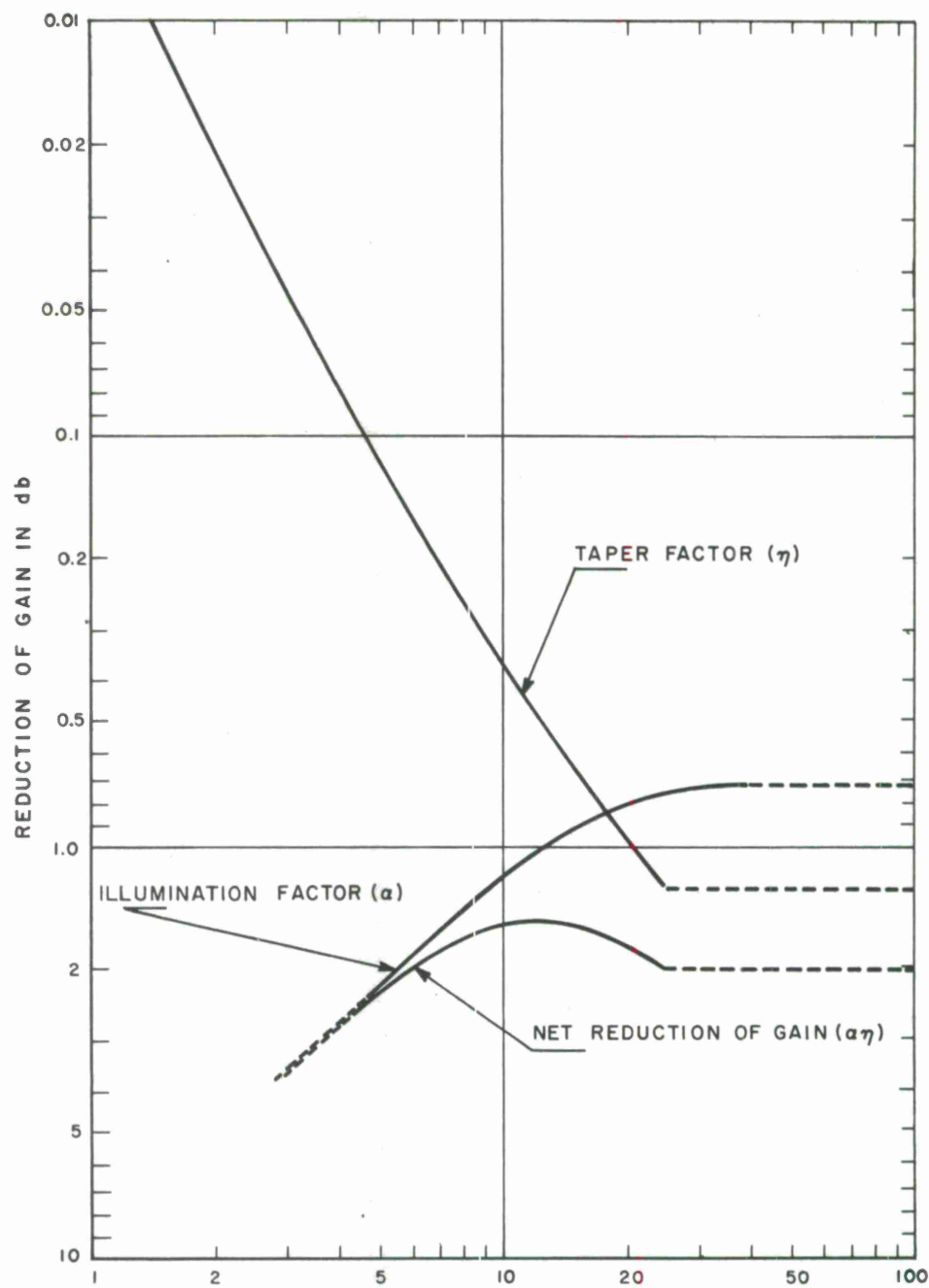


Fig. 9. Taper Factor, Illumination Factor, and Net Reduction of Gain in Decibels

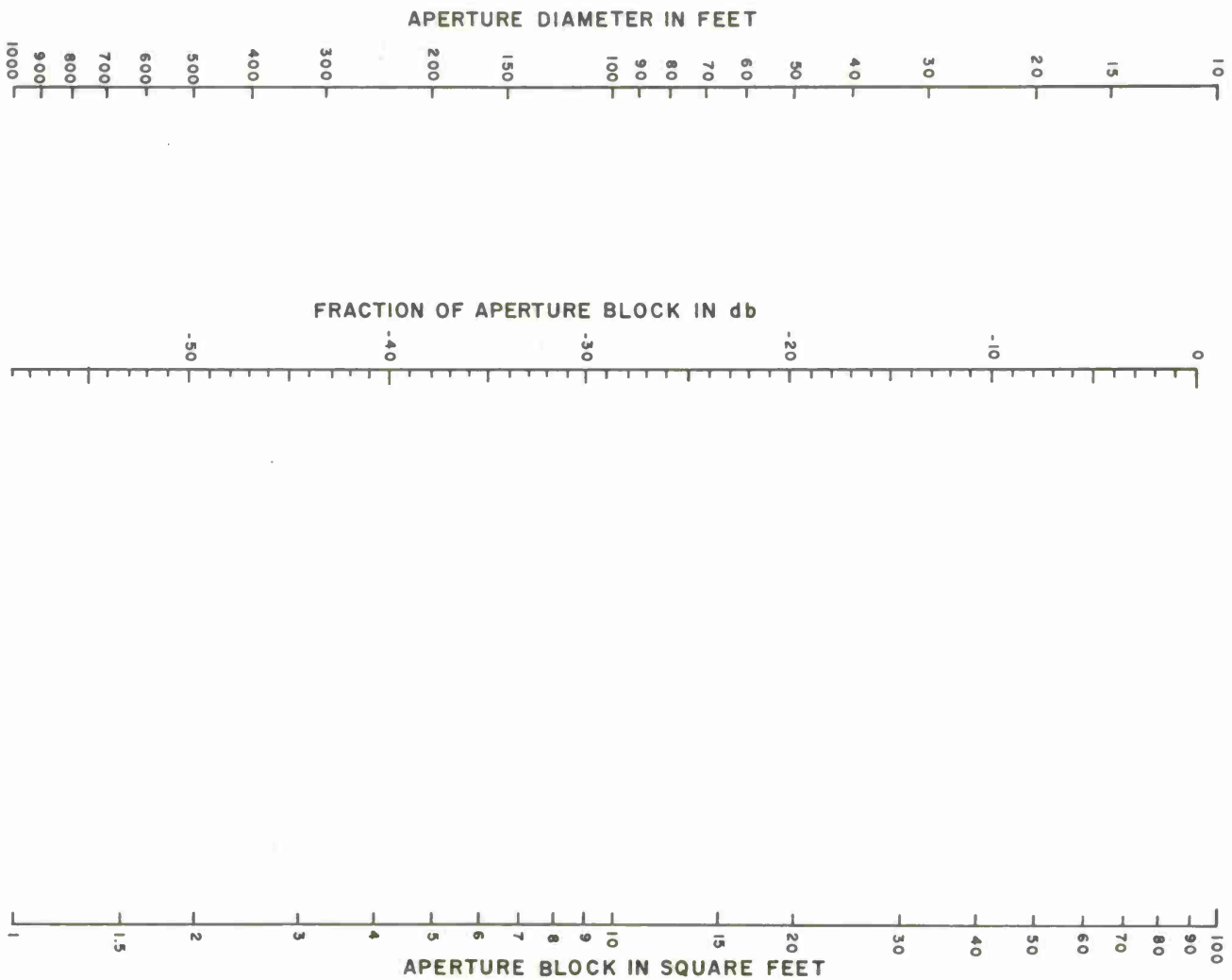


Fig. 10. Aperture Block Nomograph

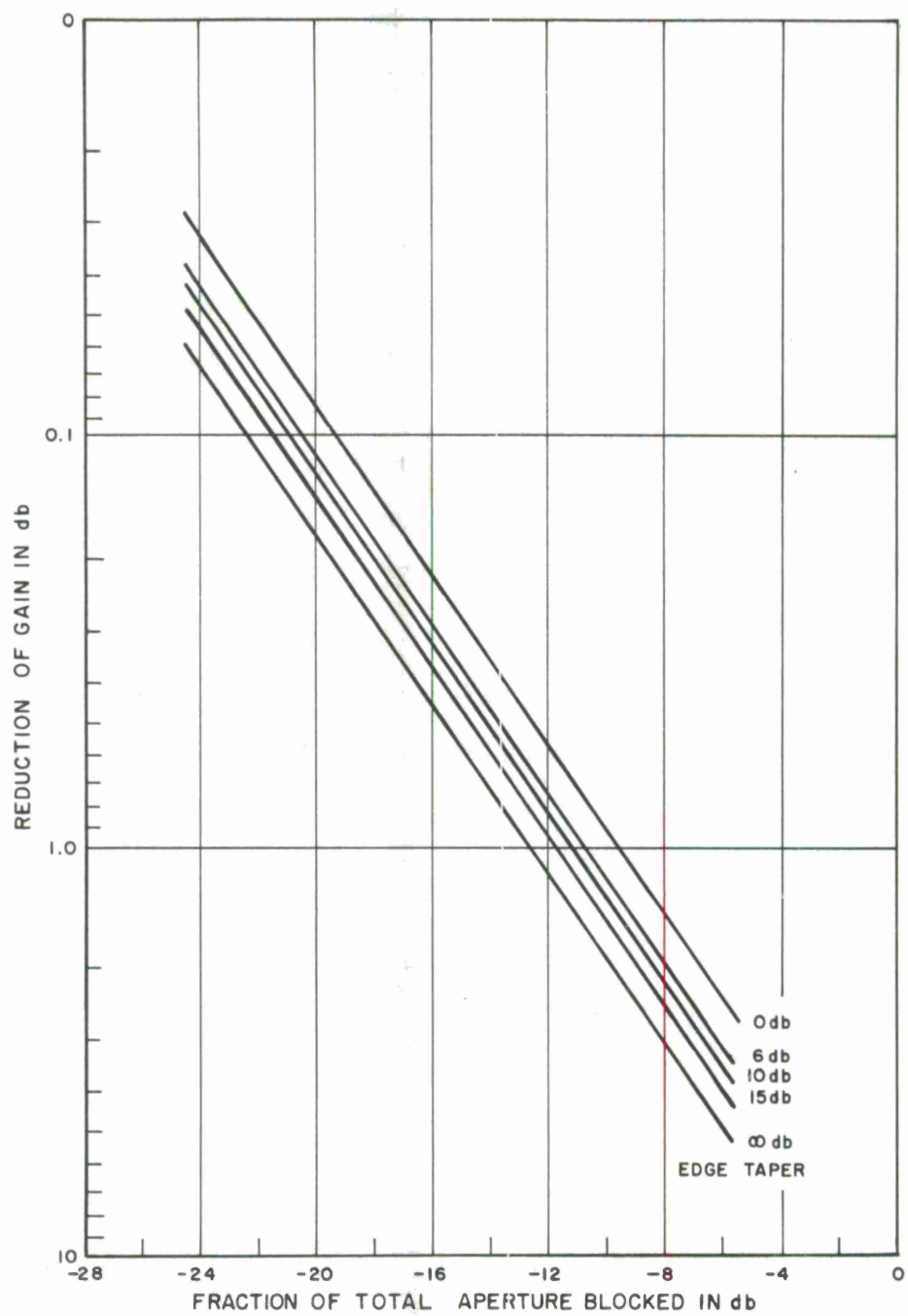


Fig. 11. Reduction of Gain Due to Aperture Block

### A Final Estimate of Peak Gain (Figures 12 through 16)

Figure 12 is a nomograph in which physical tolerance, in inches, are converted into a corresponding number of wavelengths as a function of frequency; Figs. 13, 15, and 16 give factors as a function of phase errors in fractions of wavelengths.

Figure 13 is a plot of the random phase error factor as a function of rms phase error in fractions of a wavelength. The stated rms tolerance, in inches, in the surface dimensions of the reflector can be converted to fractions of a wavelength with Fig. 12. The result should be multiplied by two. (The factor of two results because an error in the reflector surface is always approximately doubled, one error for the incoming wave, and one error for the reflected wave.) The random phase error factor, in decibels, is then read directly from Fig. 13.

Figure 14 precedes the use of Fig. 15 for use in estimating quadratic phase error effects due to axial displacement of the source. This is a typical form of the quadratic phase error.\* For a given axial displacement in fractions of a wavelength, the phase error at the edge of the aperture is  $(1 - \cos \psi)$  times the magnitude of the displacement, where  $\psi$  is a half-angle subtended by the aperture at the source and is called the angular aperture of the antenna. Figure 14 is a plot of  $\psi$  as a function of the antenna f-number (focal length/diameter), and of  $(1 - \cos \psi)$  as a function of  $\psi$ . Given the axial displacement of the primary source in inches and the f-number of the antenna, the

---

\*There are other possible sources of systematic phase errors beside an axial displacement of the primary. Figure 15 can be used to determine gain reduction from any cause that produces a quadratic phase error.

phase error at the edge of the aperture can be determined by converting inches to fractions of a wavelength with Fig. 12, and multiplying this by  $(1 - \cos \psi)$  as determined by Fig. 14.

Figure 15 gives the quadratic phase error factor,  $\gamma_2$ , in decibels, as a function of the edge displacement in wavelengths. Thus, Fig. 14 converts axial displacement into edge displacement for this purpose. Any direct, symmetrical, edge displacement can be used directly in Fig. 15. All such symmetrical error effects must be summed in terms of wavelengths before using Fig. 15.

Figure 16 is a plot of the astigmatic (bi-symmetrical) phase error in fractions of a wavelength at the maximum displacement of the edge of the aperture. Astigmatic phase error can be caused by any loading conditions that result in an elliptical warping of the reflector. The maximum amount of warping at the edge of the reflector is converted to fractions of a wavelength using Fig. 12 and the result multiplied by two. The bi-symmetrical phase error factor,  $\gamma_3$ , in decibels, can then be read directly from Fig. 16. The final estimate of peak gain is then given by:

$$G = G_m - \gamma_1 - \gamma_2 - \gamma_3 \quad (6)$$

(Both Figs. 15 and 16 have been plotted for  $\eta = 0$  db, i. e., for uniform illumination. This case produces a maximum value for symmetrical and bi-symmetrical errors, but the difference for edge tapers up to 10 db is not significant.)

#### EXAMPLE IN THE USE OF THE GRAPHS

The following basic data of an antenna are illustrative, and will be used for estimating the peak gain. The mechanical tolerances represent a "rule of thumb" design objective of 1/32 of the operating wavelength. However, this

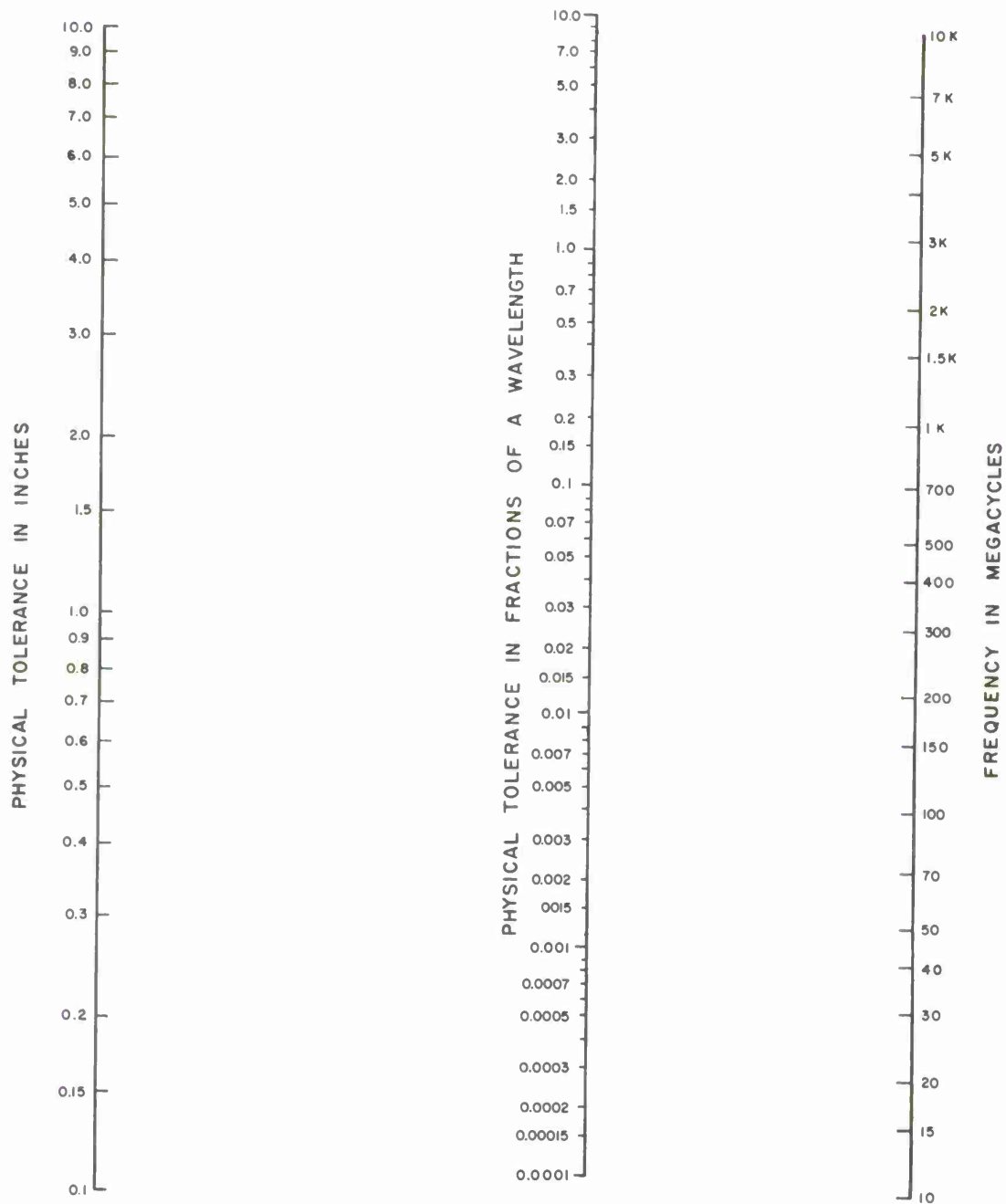


Fig. 12. Dimension Conversion Nomograph

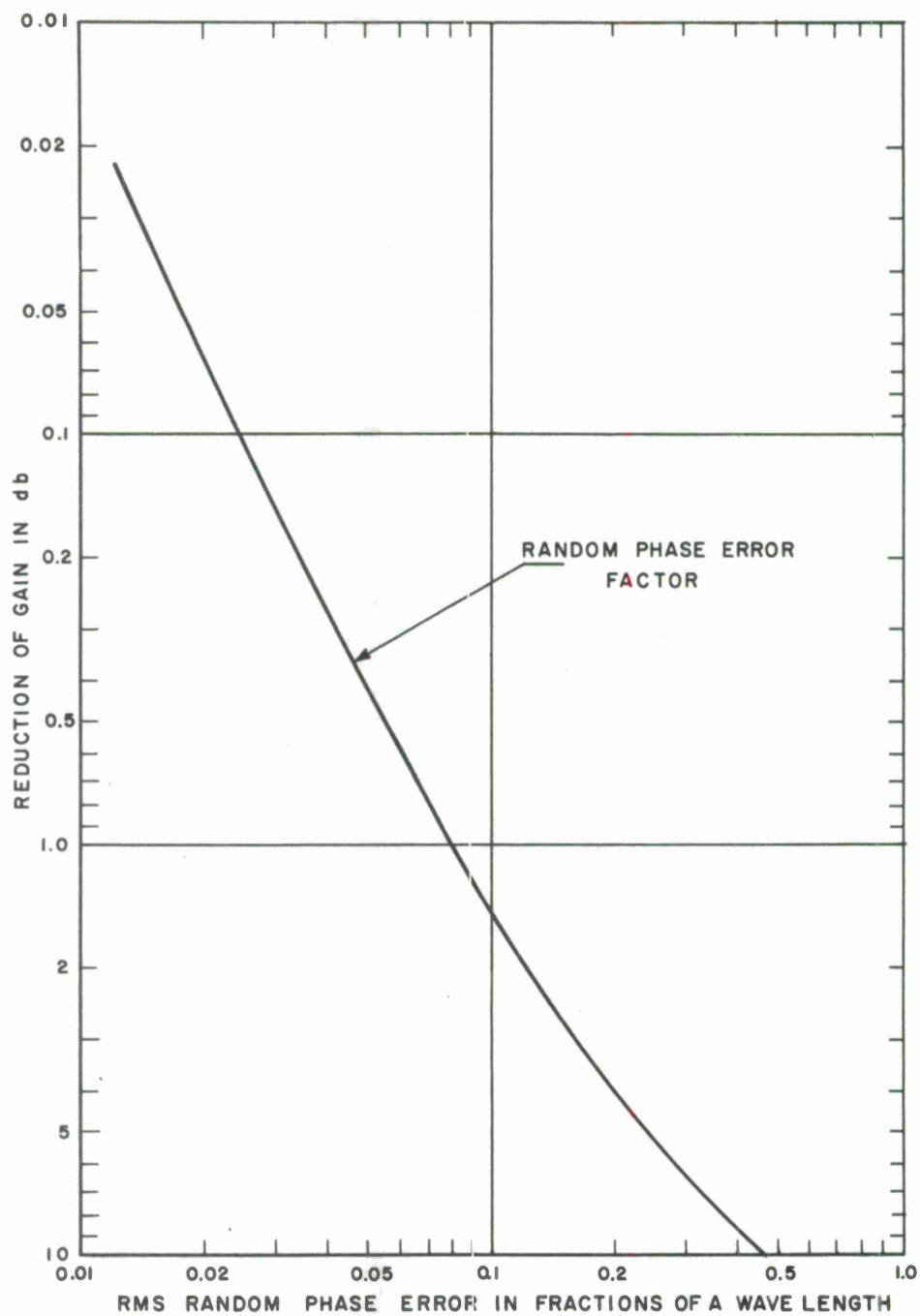


Fig. 13. Reduction of Gain Due to Random Phase Error Across a Circular Aperture

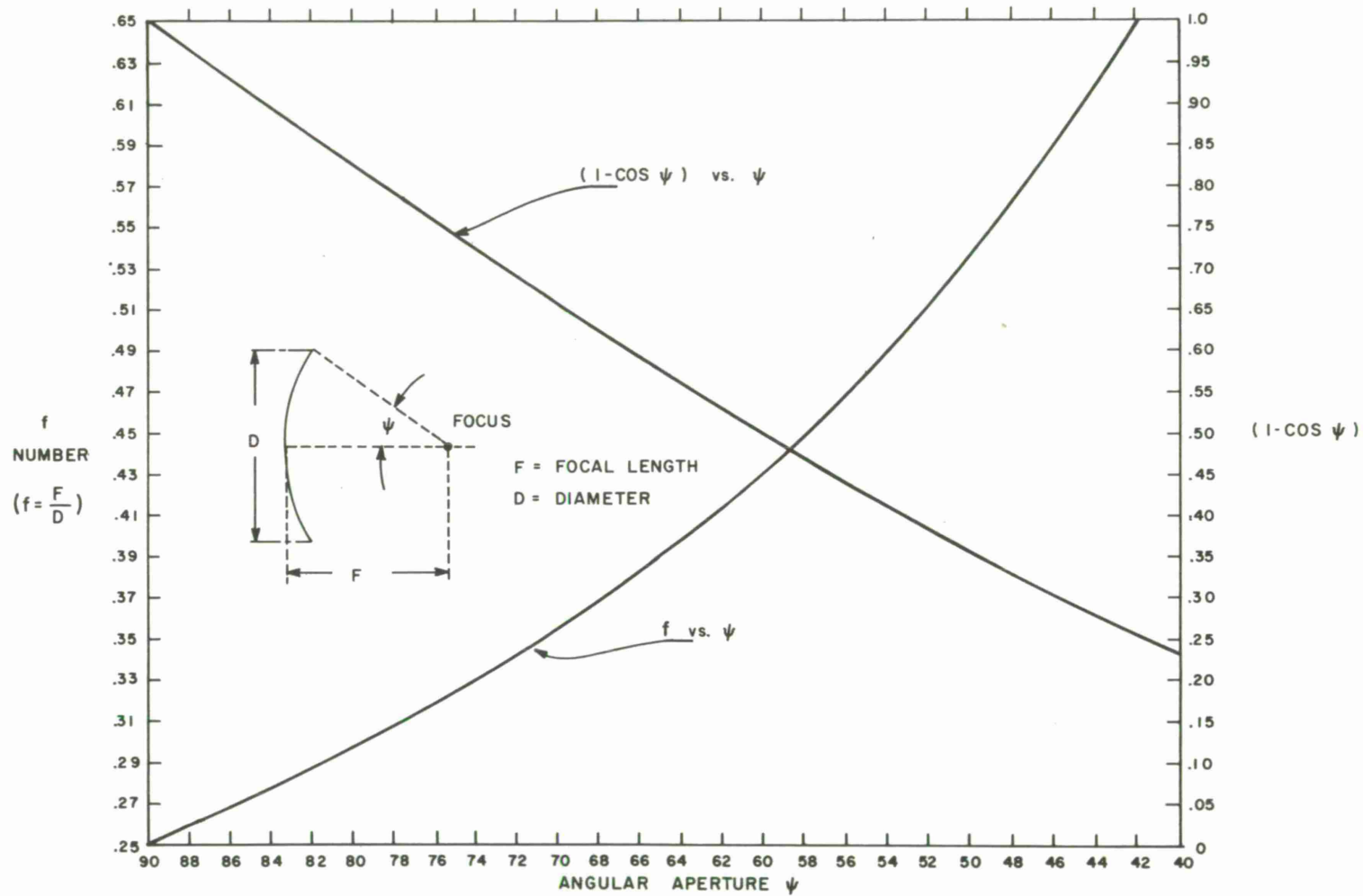


Fig. 14. F Number Versus Angular Aperture  $\psi$ , and  $(1 - \cos \psi)$  Versus  $\psi$



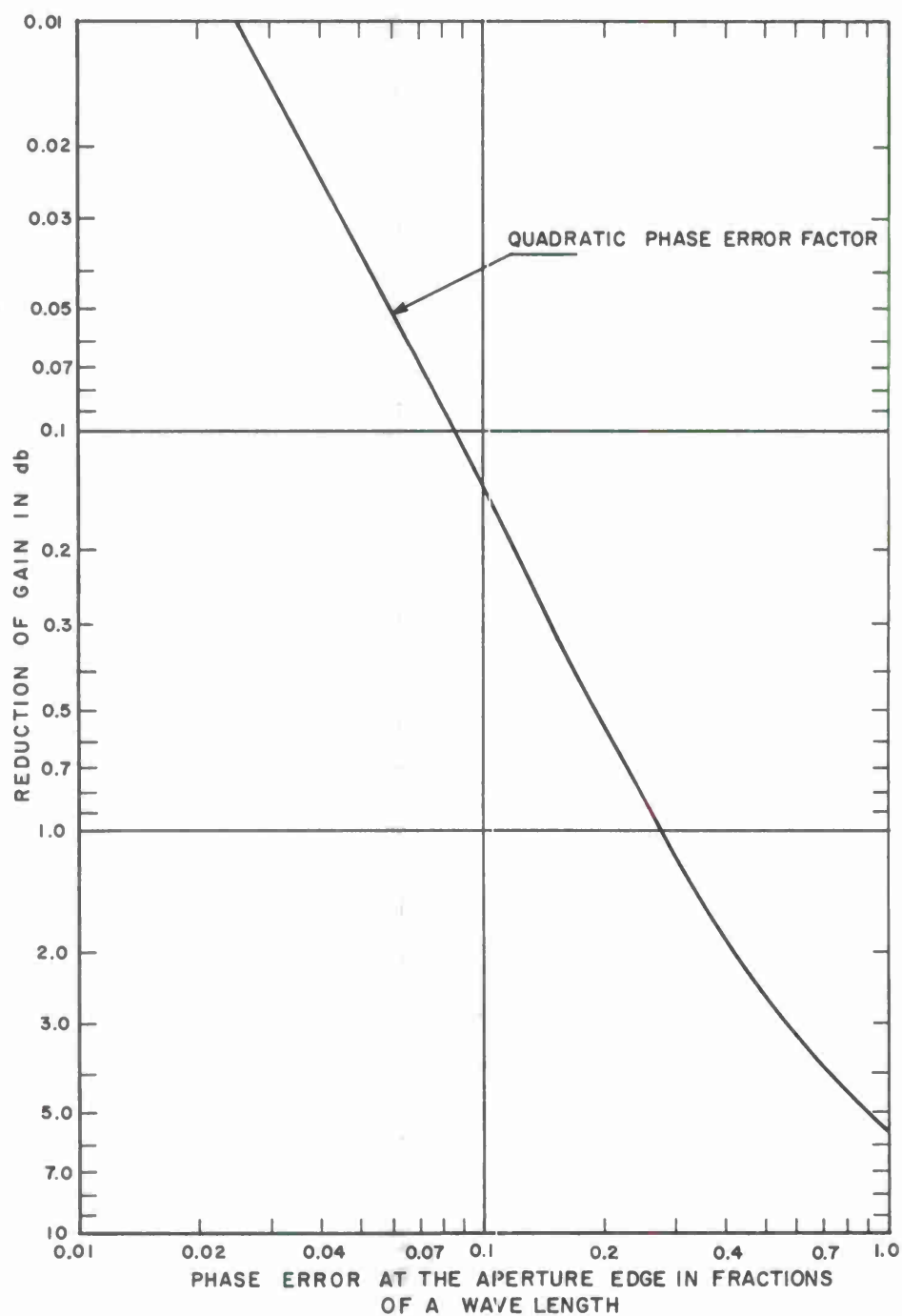


Fig. 15. Reduction of Gain Due to a Quadratic Phase Error Across a Circular Aperture

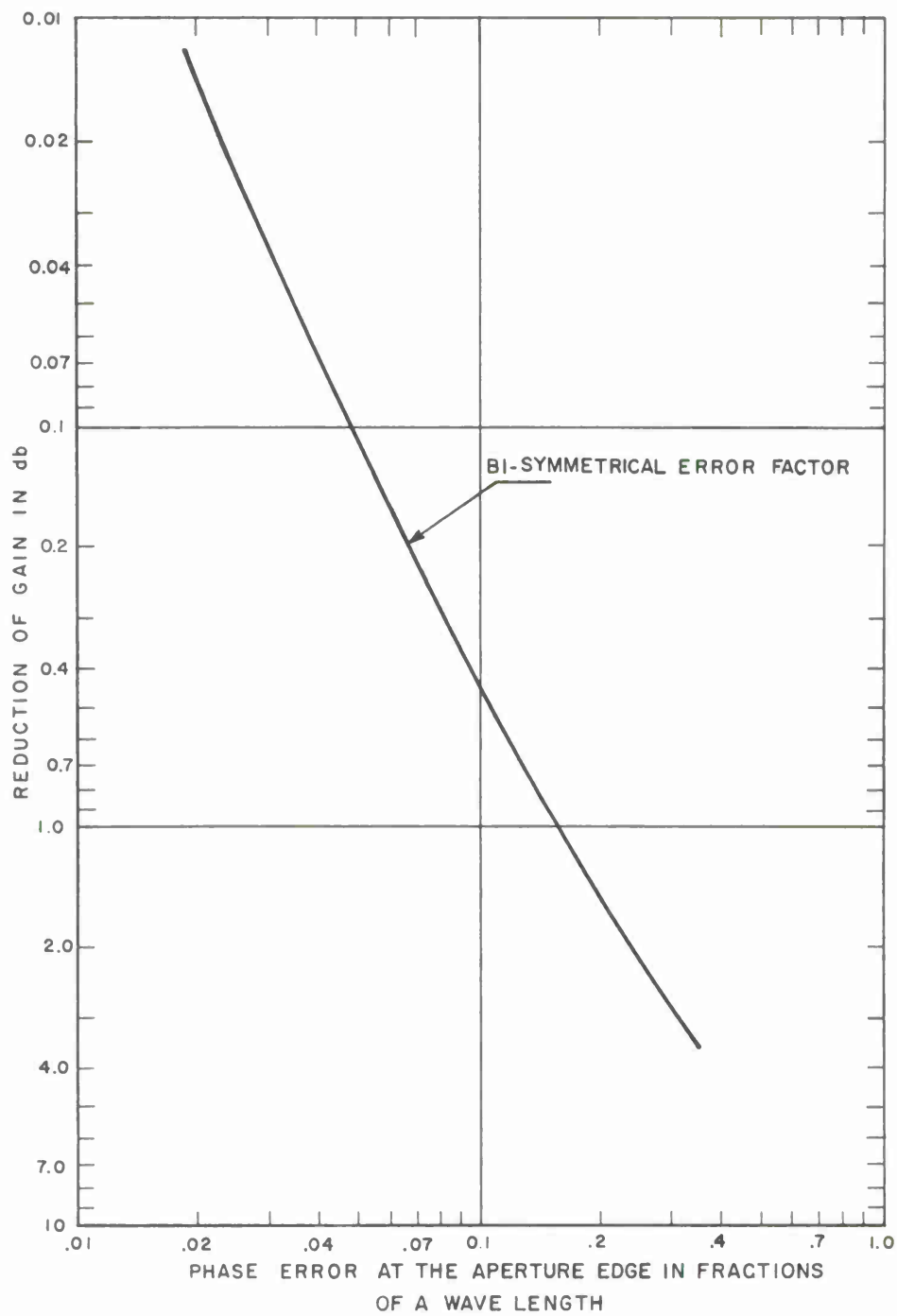


Fig. 16. Reduction of Gain Due to Astigmatic Phase Distortion

tolerance is often exceeded in fabrication, although it may be attained in the practical environment. The edge taper value chosen is close to that which produces minimum loss due to the taper factor and illumination factor; see Fig. 9.

#### Basic Antenna Data

Diameter of reflector	60 feet
Operating frequency	240 mc
Edge taper	10 db
f-number (focal length/diameter, see Appendix III)	0.4
Effective block diameter*	8.5 feet
Physical tolerances:	
Random reflector surface distortion	1.55 inches (rms)
Placement of primary source	$\pm 1.55$ inches
Symmetrical (circular) surface distortion	$\pm 1.55$ inches max.
Bi-symmetrical (elliptical) surface distortion	$\pm 1.55$ inches max.
	} at reflector edge

#### Estimates of Peak Gain

Figure 8, theoretical peak gain, $G_0$	= <u>33.2 db</u>
Figure 9, taper factor, $\eta$ , for 10-db edge taper	= 0.36 db
Figure 9, illumination factor, $\alpha$ , for 10-db edge taper	= 1.22 db

---

\*This includes mounting supports, and is typical of wide band feed systems.

Figure 10, fractional aperture block = 15.9 db

Figure 11, aperture block factor,  $\beta$ , = 0.32 db

Therefore, first estimated peak gain  $G_m$   
=  $33.2 - (0.36 + 1.22 + 0.32)$  = 31.3 db

Figure 12, conversion of tolerances  
to wavelengths,  
155 inches at 240 mc =  $0.031\lambda$   
(Twice this, for go and return,  
=  $0.062\lambda$ )

Figure 13, random phase error factor,  $\gamma_2$ , = 0.62 db

Figure 14, source-to-edge displacement  
conversion,  $\psi = 64$  degrees  
for  $f = 0.4$ ;  $(1 - \cos \psi) = 0.565$ ;  
 $(1 - \cos \psi) \times (\text{source displacement})$   
=  $0.565 \times 0.031\lambda = 0.018\lambda$ .  
(Add\* converted primary source displacement  
( $0.018\lambda$ ) to twice\*\* the  
symmetrical surface distortion  
( $0.031\lambda$ ), giving  $0.08\lambda$ , for application  
in Fig. 15.)

Figure 15, total symmetrical phase  
error factor,  $\gamma_2$ , = 0.09 db

Figure 16, (use twice bi-symmetrical  
edge distortion [ $0.062\lambda$ ]  
Bi-symmetrical Phase Error  
Factor,  $\gamma_3$ , = 0.16 db

Therefore, final estimated peak gain,  
 $G = 31.3 - (0.62 + 0.09 + 0.16)$  = 30.4 db

---

\* This conservative approach is justified because we are making an operational estimate of antenna gain.

\*\*A source displacement error occurs once in a given ray path; whereas, a reflector displacement occurs twice; that is, in the incident and reflected paths.

## THE EFFECT OF OPERATING FREQUENCY ON ANTENNA GAIN AND SYSTEM COSTS

The example taken in the previous section could be typical of missile range telemetry antennas now operating in the normal VHF telemetry band, and shows that the reduction in theoretical gain can be 3 db, assuming that the design tolerances of 1/32 of the operating wavelength can be maintained under operating environmental conditions. Of this 3 db, 2 db is contributed by the taper, illumination and aperture block factors, and these are independent of operating frequency.

If it is desired to use this antenna at higher frequencies, such as the new 2200-2300 mc band, the loss of gain due to phase error factors could be much greater, for the same mechanical tolerances and displacements. For example, at 2200 mc,

Random phase error factor $\gamma_1$ ,	= 11 db
Total symmetrical phase error factor, $\gamma_2$ ,	= 4 db
Bi-symmetrical phase error factor, $\gamma_3$ ,	= 7 db

These total 22 db, as compared to 1 db at 240 mc, for the original assumption of 1.55-in. mechanical and deformation tolerances. The theoretical gain,  $G_0$ , is higher, at 52.3 db, but so is the free-space path loss over a given link, and by exactly the same amount as the increase in antenna gain. Hence, the system margin would be reduced by the increase in the phase error losses; that is, by 21 db.

The only remedy for this situation is to set the design tolerances for manufacture and environment at the same wavelength fraction as for the 240-mc case; that is, the design tolerances for each type of mechanical error or deformation must be 0.17-in., approximately, 3/16-in. While it is possible to manufacture, and even to set up these large antennas on site, to such tolerances

(by the use of segmental adjustment features), it is doubtful if they can be maintained in a severe environment. Furthermore, the on-site checking of the deformation is not an easy or cheap matter, and usually cannot be done properly under the environmental conditions. For example, it is clearly difficult, if not impossible, to do such checks for a large ship-mounted antenna under storm conditions.

The design and use of large telemetry antennas at high operating frequencies is, therefore, a complicated matter that will inevitably result in greater costs somewhere in the system.

#### IV GENERAL CONCLUSIONS

This report has shown that the practical peak gain of a parabolic antenna, designed for the conventional tolerances of  $1/32$  of the operating wavelength, can be at least 3db less than the theoretical gain; that is, the "efficiency" is 50 percent or less.

The use of large antennas, such as the 60-foot dishes used for missile range telemetry that have been designed for the VHF band, at higher frequencies, may introduce loss of overall system gain due to phase errors. For example, the net phase error loss of 1db at 240 mc, for mechanical tolerances of  $1/32$  of the operating wavelength, rises to 22db at 2200 mc, and this is a system loss.

On the other hand, the design of a 60-foot dish with the maximum tolerable  $1/32$  wavelength mechanical tolerances and deformations for operation at 2200 mc, means maximum tolerances of  $3/16$ -in. in surface, in aperture edge location, and in source (feed) location, under all operating and environmental conditions.

Either way, the design and use of large parabolic antennas at relatively high frequencies is difficult and costly, if the normal system margins are to be maintained.

In particular, the replacement of a smaller dish by a larger, in conditions where wide-band operation is desired under severe environmental conditions, may result in a degraded system performance, unless detailed attention is paid to the gain modifying factors discussed in this report in its design, test, and use.

Finally, it should be noted that nothing has been said about the causes of mechanical deformation (such as temperature, wind, ice, etc.), because this is strictly a design problem. But it should be obvious that mechanical structures



of between 30- and 60-foot linear dimensions, rotatable in azimuth and elevation, must deform on the order of fractions of an inch when subjected to the usual range of weather conditions.

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## APPENDIX I

### DEPENDENCE OF PEAK GAIN ON AMPLITUDE DISTRIBUTION (UNIFORM PHASE)

#### PEAK APERTURE GAIN<sup>[3]</sup> (General Form)

A general gain expression valid for a uniform phase distribution and any amplitude distribution is given by:

$$G = \frac{4\pi}{\lambda^2} \frac{\left| \int_A F(x, y) dx dy \right|^2}{\int_A \left| F(x, y) \right|^2 dx dy}, \quad (I-1)$$

where  $F(x, y)$  is the magnitude of the electric field distribution across the aperture as a function of  $x$  and  $y$ , rectangular coordinates of a point in the aperture plane and  $\int_A = \iint$ .

For a uniform amplitude distribution, Eq. (I-1) reduces to Eq. (I-2), for an irregularly shaped plane aperture of area  $A$ ;

$$G_0 = \left( \frac{4\pi A}{\lambda^2} \right), \quad (I-2)$$

and it is seen that gain is proportional to the area of the aperture.

For a uniform phase and amplitude distribution, the aperture gain of a circular aperture of diameter  $D$  is given by:

$$G_0 = \left( \frac{\pi D}{\lambda} \right)^2,$$

where  $\lambda$  is the wavelength.

## APERTURE EFFICIENCY FACTOR (TAPER FACTOR), $\eta$

If the gain of an aperture with a nonuniform amplitude distribution is considered to be the gain of uniformly illuminated aperture multiplied by an aperture efficiency factor,  $\eta$ , then from Eq. (I-1) we have:

$$\eta = \frac{1}{A} \frac{\left| \int_A F(x, y) \, dx dy \right|^2}{\int_A |F(x, y)|^2 \, dx dy} \quad (\text{I-4})$$

If the amplitude distribution is not radially constant, then it will be presumed to be symmetrical about the axis of the parabola. In general, the amplitude distribution vs. radius will include a constant term and terms of higher order. However, if computations are made on this generalized basis (i. e., by the use of a power series summation, with general coefficients), then it is found that the error due to not using terms of higher order than the square is insignificant. Hence, in this work a parabolic distribution plus a constant term, across the circular aperture, will be used as follows:

$$F(r) = E_0 \left[ 1 - \left( \frac{2r}{D} \right)^2 (1 - a) \right], \quad (\text{I-5})$$

where  $E_0$  is the amplitude of the field at the center of the aperture, ( $a E_0$ ) is the amplitude at the edge of the aperture, and  $D$  is the aperture diameter;  $a$  is fractional edge taper. With Eq. (I-4) written in polar coordinates, direct substitution of Eq. (I-5) into Eq. (I-4) and integration gives:

$$\eta = \frac{1}{1 + \frac{1}{3} \left( \frac{1 - a}{1 + a} \right)^2} \quad (\text{I-6})$$

The results of an evaluation of this equation are shown in Table I-1 for edge tapers from 1 to  $\infty$  db and plotted on Fig. 9.

Table I-1

Aperture Efficiency Factor as a Function of Edge Taper  
for a Parabolic Aperture Distribution

Edge Taper (db)	Fractional Edge Taper (a)	Aperture Efficiency ( $\eta$ )	
		<u><math>\eta</math></u>	<u>Loss in db</u>
1	0.892	0.998	
2	0.795	0.995	0.02
3	0.707	0.991	0.04
4	0.631	0.984	0.07
5	0.563	0.975	0.11
6	0.510	0.965	0.15
7	0.446	0.954	0.20
8	0.398	0.940	0.27
9	0.355	0.930	0.31
10	0.316	0.918	0.37
12	0.251	0.894	0.49
15	0.178	0.860	0.65
20	0.100	0.817	0.88
25	0.006	0.751	1.24
$\infty$	0	0.750	1.25

### ILLUMINATION EFFICIENCY FACTOR, ( $\alpha$ )

When a fraction,  $\alpha$ , of the total power radiated by the source is intercepted by aperture, the peak aperture gain,  $G_A$ , of the antenna is given by:

$$G_A = \frac{4\pi P_0}{\alpha P_T} \quad (I-7)$$

where

$P_0$  is the peak power density, and

$P_T$  is the total power radiated by the source.

The actual gain,  $G$ , of the entire antenna is given by:

$$G = \frac{4\pi P_0}{P_T} \quad (I-8)$$

Therefore,

$$G = \alpha G_A, \quad (I-9)$$

and the antenna gain is equal to the aperture gain times the fraction of the total power radiated that is intercepted by the aperture. This fraction,  $\alpha$ , cannot exceed the fraction of power radiated in the main lobe of the primary source. (For this case, a maximum of 85 percent of the total power radiated is contained in the main lobe of the primary.)

In Table I-2, the values of  $\alpha$  as a function of edge taper for paraboloids illuminated by small horns are taken from graphical integrations made by Berkowitz,<sup>[8]</sup> and appears in Thourel's text,<sup>[5]</sup> page 266. It is these results that have been plotted in Fig. 9.

Table I-2

Illumination Efficiency Factor as a Function of Edge Taper  
for Paraboloids Illuminated by Small Horns

Edge Taper (db)	Illumination Efficiency Factor ( $\alpha$ )	Loss (db)
8	0.71	1.49
10	0.75	1.25
12	0.79	1.02
15	0.81	0.91
20	0.83	0.81
25	0.84	0.75
$\infty$	0.85	0.71

#### APERTURE BLOCK FACTOR, $\beta$

The feed structure and supporting numbers of a paraboloidal antenna block part of the radiated energy at the center of the aperture. This results in a region of low intensity in the aperture distribution. The effect of this region of low intensity may be calculated by resolving the aperture illumination into two separate radiating apertures, one the aperture distribution of the unblocked aperture, and the other an out-of-phase component which, when added to the first, would result in the actual illumination distribution encountered. The far-field strength is calculated for each component separately and then added. In a region about the main lobe and the first side lobes of the antenna pattern the result is a reasonable approximation of the field strength.<sup>[9]</sup>

The aperture block factor,  $\beta$ , can therefore be expressed in terms of the far field strengths as

$$\beta = 20 \log \left[ 1 - \frac{E_B}{E_A} \right], \quad (I-10)$$

where

$E_B$  is the field strength due to the blocking aperture, and

$E_A$  is the field strength due to the unblocked aperture.

The field strength,  $E$ , is equal to the square root of the product of the aperture gain and the power crossing the aperture. Peak aperture gain is given by Eq. (I-1) and the power crossing the aperture is proportional to  $\int_A |F(x,y)|^2 dx dy$ , where  $F(x,y)$  is the field distribution across the aperture. The ratio,  $E_B/E_A$ , is, therefore, given by

$$\frac{E_B}{E_A} = \frac{\int_B F(x,y) dx dy}{\int_A F(x,y) dx dy}, \quad (I-11)$$

where

$A$  is the area of the unblocked aperture, and

$B$  is the area of the blocking aperture.

For the aperture distribution given by Eq. (II-5),

$$\frac{E_B}{E_A} = \left[ \frac{2}{(1+a)} - \frac{(1-a)}{(1+a)} \left( \frac{B}{A} \right) \right] \left( \frac{B}{A} \right)^*. \quad (I-12)$$

---

\*This formula is exact when  $A$  and  $B$  are circular areas. However, it is accurate as long as  $B \ll A$ , no matter what shape  $B$ .

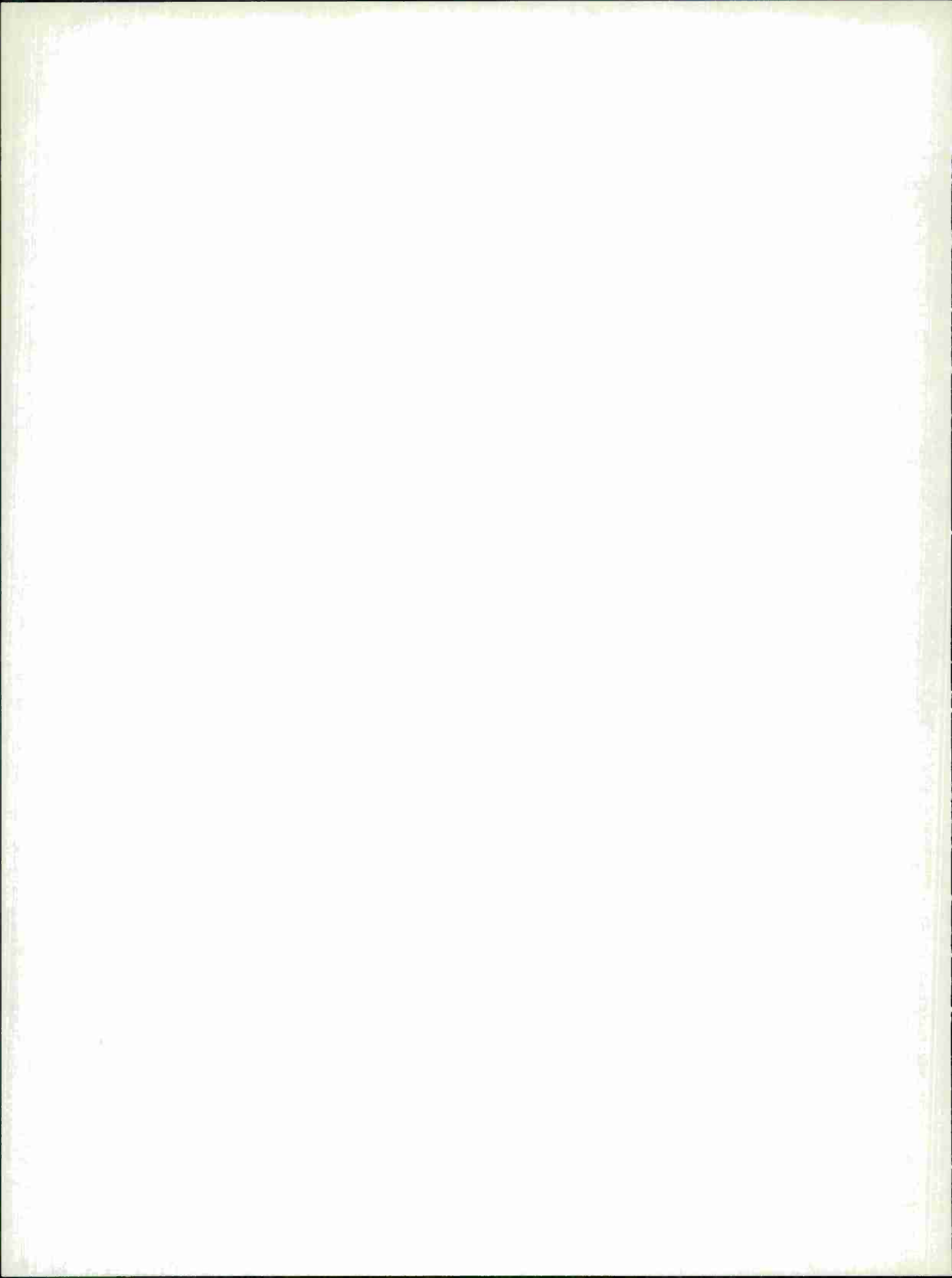


Substitution of Eq. (I-12) into Eq. (I-10) gives

$$\beta = 20 \log \left[ 1 - \left( \frac{B}{A} \right) \right] \left[ 1 - \frac{(1-a)}{(1+a)} \left( \frac{B}{A} \right) \right] . \quad (I-13)$$

An interesting consequence of this formula is the fact that  $\beta$  for an infinite edge taper ( $a = 0$ ) is exactly twice  $\beta$  for a zero edge taper ( $a = 1$ ), as substitution of these values of  $a$  in Eq. (I-13) shows:

$$\beta (a = 0) = 2\beta (a = 1) . \quad (I-14)$$



## APPENDIX II

### ANALYSIS OF LOSS OF GAIN DUE TO PHASE ERRORS

#### GENERAL PHASE ERROR FACTOR [7] \*

For any given electric field distribution across the aperture of an antenna, it is possible to define a mean electric field as

$$E_m = \frac{1}{A} \int_A E_A dA, \quad (\text{II-1})$$

where

$E_A$  is the magnitude of the electric field in an element of area  $dA$ , and

$A$  is the area of the aperture.

If the magnitude of the electric field in the plane of a circular aperture is circularly symmetrical, it can be expressed as a function of the radius of the aperture. Its mean value is then

$$E_m = \frac{1}{\pi R^2} \int_0^R E(r) 2\pi r dr, \quad (\text{II-2})$$

where

$R$  is the radius of the aperture,

$2\pi r dr$  is an element of area within which  $E(r)$  is constant for any given value of  $r$ , and

$\pi R^2$  is the area of the aperture.

---

\*Bracewell [7] calls this directivity achievement factor.

The E-field across the aperture can also be written as an error function in terms of a design error factor,  $\epsilon$ , and the mean value,  $E_m$  :

$$E = E_m + \epsilon E_m . \quad (\text{II-3})$$

The variance, or mean square value of  $\epsilon$ , is defined by an integral of the same form as Eq. (II-1):

$$\text{var } (\epsilon) = \frac{1}{A} \int_A \epsilon \epsilon^* dA . \quad (\text{II-4})$$

Again, for a circularly symmetrical field distribution across a plane circular aperture,  $\epsilon$  is also a function of the radius, and Eq. (II-4) can be written as:

$$\text{var } (\epsilon) = \frac{1}{\pi R^2} \int_0^R |e(r)|^2 2\pi r dr , \quad (\text{II-5})$$

where  $R$  is the radius of the aperture. The aperture efficiency can now be written in terms of  $\text{var } (\epsilon)$ . Substituting Eq. (II-3) into Eq. (I-4) of Appendix I, we have:

$$\eta = \frac{1}{A} \frac{\left| \int_A (1 + \epsilon) dA \right|^2}{\int_A |1 + \epsilon|^2 dA} . \quad (\text{II-6})$$

The numerator of Eq. (II-6) is equal to  $A^2$ . The denominator can be evaluated by writing:

$$\begin{aligned} |1 + \epsilon|^2 &= (1 + \epsilon)(1 + \epsilon^*) \\ &= 1 + \epsilon + \epsilon^* + \epsilon \epsilon^* . \end{aligned}$$

Since  $\int_A \epsilon da = \int_A \epsilon^* da = 0$  and  $\text{var}(\epsilon) = \frac{1}{A} \int_A \epsilon \epsilon^* dA$ , from Eq. (II-4),

then

$$\eta = \frac{1}{1 + \text{var}(\epsilon)} . \quad (\text{II-7})$$

If the actual field distribution differs by some small amount so that the actual error factor is not  $\epsilon$  but some slightly different error factor,  $\epsilon'$ , then the aperture efficiency is really:

$$\eta = \frac{1}{1 + \text{var}(\epsilon')} . \quad (\text{II-8})$$

The ratio  $\eta'/\eta$  is the definition of the phase error factor.

$$\gamma = \frac{\eta'}{\eta} \frac{1 + \text{var}(\epsilon)}{1 + \text{var}(\epsilon')} . \quad (\text{II-9})$$

The difference between the two error factors is

$$\epsilon_r = \epsilon' - \epsilon .$$

In terms of  $\epsilon_r$ , the phase error factor is

$$\gamma = \frac{1}{1 + \eta (\text{var} \epsilon_r + \langle \epsilon \epsilon_r^* \rangle + \langle \epsilon_r^* \epsilon \rangle)} , \quad (\text{II-10})$$

where

$$\langle \text{----} \rangle = \frac{1}{A} \int_A \text{----} dA .$$

Equation (II-10) is important because it shows how the gain is reduced by small variations,  $\epsilon_r$ , from a previously determined or desired aperture distribution as expressed by  $\epsilon$ .

## RANDOM PHASE ERROR FACTOR

Equation (I-4) assumes a uniform phase distribution and a specified amplitude distribution. For such a situation,  $\epsilon$  is a real function. However, for pure phase errors, the function  $\epsilon_r = \epsilon' - \epsilon$  is purely imaginary. Under these conditions,  $\epsilon = \epsilon^\bullet$  and  $\epsilon_r = -\epsilon_r^\bullet$ . Consequently,

$$\langle \epsilon \epsilon_r^\bullet \rangle + \langle \epsilon_r^\bullet \epsilon_r \rangle = \langle -\epsilon \epsilon_r \rangle + \langle \epsilon \epsilon_r \rangle = 0,$$

and

$$\gamma = \frac{1}{1 + \eta \text{ var } (\epsilon_r)} . \quad (\text{II-11})$$

For a pure phase error,  $\varphi$ , in radius,  $|\epsilon_r| = |1 + \epsilon| \varphi$ , provided  $\varphi$  is small.  $\text{Var } (\epsilon_r)$  is, therefore, the mean square value of the weighted phase error over the aperture;  $|1 - \epsilon|$  is the weighting factor.

If  $\varphi$  is an rms phase tolerance across the aperture for random phase errors, the expected or mean value of  $\gamma$  is determined using a mean value of  $\text{var } (\epsilon_r)$  where

$$\text{var } (\epsilon_r) = \frac{\varphi^2}{\pi R^2} \int_0^R |1 + \epsilon|^2 2\pi r dr . \quad (\text{II-12})$$

For a parabolic field distribution

$$\epsilon = \left( \frac{1 - a}{1 + a} \right) \left[ 1 - 2 \left( \frac{r}{R} \right)^2 \right], \quad (\text{II-13})$$

where  $a$  is the edge taper. Integrating Eq. (II-12) for  $\epsilon$  given by Eq. (II-13), yields

$$\text{var } (\epsilon_r) = \left[ 1 + \frac{1}{3} \left( \frac{1-a}{1+a} \right)^2 \right] \varphi^2. \quad (\text{II-14})$$

Since  $\eta = \left[ 1 + \frac{1}{3} \left( \frac{1-a}{1+a} \right)^2 \right]^{-1}$ , for the same distribution, then

$$\gamma = \frac{1}{1 + \varphi^2} \quad (\text{II-15})$$

It is this equation that is used to plot Fig. 13.

#### THE QUADRATIC PHASE ERROR FACTOR

If the phase varies as the square of  $r$  to a maximum  $\varphi_0$  at the extreme edge of the circular aperture, the phase at any point in the aperture can be expressed as:

$$\varphi = \varphi_0 \left( \frac{r}{R} \right)^2, \quad (\text{II-16})$$

where  $R$  is the maximum radius of the aperture. For a uniform amplitude distribution,  $\eta = 1$ , Eq. (II-11) reduces to:

$$\gamma = \frac{1}{1 + \text{var } (\epsilon_r)}. \quad (\text{II-17})$$



The weighting factor for  $\eta = 1$  is  $|1 + \epsilon| = 1$  and

$$\begin{aligned}
 \text{var } (\epsilon_r) &= \langle \varphi^2 \rangle - \langle \varphi \rangle^2 \\
 &= \frac{1}{\pi R^2} \int_0^R \varphi^2 2\pi r dr - \left[ \frac{1}{\pi R^2} \int_0^R \varphi 2\pi r dr \right]^2 \\
 &= \frac{2\varphi_0^2}{R^6} \int_0^R r^5 dr - \left[ \frac{2\varphi_0}{R^4} \int_0^R r^3 dr \right]^2 \quad (\text{II-18}) \\
 &= \frac{1}{3} \varphi_0^2 - \frac{1}{4} \varphi_0^2 = \frac{\varphi_0^2}{12} .
 \end{aligned}$$

Substituting into Eq. (II-17), we have

$$\gamma = \frac{1}{1 + (\varphi_0^2/12)} . \quad (\text{II-19})$$

If  $\eta < 1$ , the combination of the weighting factor and  $\eta$  reduces the value of  $\eta \text{ var } (\epsilon_r)$ , with a resultant higher value of  $\gamma$ . The loss of gain is therefore reduced. Figure 15 is a plot of Eq. (II-19).

#### THE ASTIGMATIC PHASE ERROR FACTOR

For an astigmatic phase error resulting in a maximum phase error of  $\varphi$  radians at the edge of the aperture, [7]

$$\gamma = \frac{1}{1 + \frac{1}{4} \eta \varphi^2} . \quad (\text{II-20})$$

This is the function used in plotting Fig. 16.

## APPENDIX III

### THE F-NUMBER AND SOURCE RADIATION PATTERNS

In the "Example in the Use of the Graphs" in Section III, note that the f-number (focal length/diameter) is quoted as a given parameter, in accordance with usual practice. Since the choice of this value will clearly affect the phase error computations, a few comments on the possible range of f-numbers are relevant.

The angle between the first nulls of the source radiation pattern is the largest angle that can be subtended by the aperture at the source. The source is then at the minimum permissible distance from the center of the aperture. The maximum value of this angle ( $2\psi$  in Fig. 14) is unlikely to exceed 180 degrees, for a practical source radiation pattern. The source is then in the aperture plane and the corresponding (minimum) f-number is 0.25. This condition, however, would also give an infinite edge taper and a high loss due to poor use (illumination) of the reflector. Hence, the practical minimum f-number is about 0.3.

It is, of course, permissible to use any higher f-number that gives the desirable 10 db edge taper from the main lobe of the source. But, in general, the larger the f-number the greater the mechanical support difficulty, and the more attention has to be paid to the side-lobe pattern of the source.

Hence, in general, it is rare to find f-numbers larger than 0.4 used without the aid of intermediate reflectors. However, a careful inspection of Fig. 14 shows that numbers up to 0.6 do have advantages in reducing the phase error losses of axial displacements in the source (provided the increased support length does not itself provide additional displacement due to temperature changes).

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13. ABSTRACT The characteristics that affect the practical operating gain of parabolic reflector-type antennas are discussed, not for design considerations, but to analyze for the systems engineer some of the factors involved in estimating practical performance under arbitrarily set conditions. The effects of structural design, mechanical tolerances and deformations, illumination, and other considerations on the gain of large antennas operating at relatively high frequencies are examined, using theory, graphic data, and an analysis of a typical case using a conventional 60-foot telemetry antenna as an example. The possible system performance degradation resulting from the use of existing large antennas in the new 2200 to 2300-mc telemetry band is discussed.			



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